Relating shear strength of unsaturated soils with capillary water retention curve

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Abstract. This paper proposes a new water retention model for unsaturated soils, which takes into account capillary condensation of adsorbed water. In the proposed water retention model, the degree of saturation of a soil is separated into that based on capillary water and that based on adsorbed water. Through the analysis of a partially saturated two-cylinder system, a new shear strength criterion for unsaturated soils is proposed, in which only the degree of saturation based on capillary water contributes to the variation of shear strength with suction. The proposed shear strength criterion is justified against thermodynamic principles. The proposed strength criterion is compared against existing criteria in the literature, which shows that it provides a much improved prediction of the experimental data, for a wide range of suction values.

1 Introduction

A number of shear strength criteria for unsaturated soils have been proposed in the literature during past three decades or so. It is often recognised that the shear strength of an unsaturated soil is closely related to the suction or the degree of saturation in the soil. Therefore, the degree of saturation that can be obtained from the water retention curve for a given suction has been widely embedded into strength criteria for unsaturated soils, such as [1]; [2]; [3] and [4]; etc.

With a given suction, water retained in soil pores due to different processes contributes differently to the shear strength of the soil. Pore water in an unsaturated soil can be divided into two categories: capillary water and adsorbed water. Only the capillary water contributes to the shear strength. Capillary water exists among soil particles and the pressure of capillary water affects the contact stress (σ') among soil particles [3], [5]. Compared with capillary water, the contribution from adsorbed water to the shear strength of the soil is very limited [6-9]. In principle, this is because adsorbed water wraps the surface of each soil particle and the thus pressure of adsorbed water almost does not affect the contact stress (σ') among soil particles.

On the other hand, water retention equations for unsaturated soils, such as [10] and [11], are commonly based on simple regression of experimental data, and overlook the physical mechanisms behind. These water retention equations usually assume that all pore water exists as capillary water by simplifying soil pores to capillary cylinders. No specific surface area is explicitly considered in these equations and hence the existence of adsorbed water is ignored. When the degree of saturation is embedded in the shear strength criteria [3, 12], it leads to significant overestimation of shear strength of unsaturated soils, especially for fine-grained soils and high suction values [13]. Using the effective degree of saturation to replace the degree of saturation in shear strength criteria [2, 8, 9] can somewhat improve the prediction, but still overestimates the shear strength for a wide range of suction values [13]. The use of the effective degree of saturation somewhat acknowledges the existence of adsorbed water, as the residual water content, that exists mostly in form adsorbed water, comes into play. However, in the definition of the effective degree of saturation, the volume of residual water or microscopic water (i.e., the water attached to the particles or trapped in the micro-pores) is usually assumed to be either independent of suction [2, 9, 14-16] or increasing monotonically along with the decrease of suction [8, 17-19]. The assumption of constant adsorbed water conflicts with surface physics principles [5]. Besides, adsorbed water cannot always increase along with the decrease of suction because of capillary condensation [20]. Other strength criteria use an additional fitting parameter to improve their predictions [1, 4] According to a comparative study by [13], shear strength criteria with additional fitting parameters usually fit observed data better, but the challenge is to determine these additional parameters.

The objective of this study is to derive the shear strength criterion for unsaturated soils by considering that pore water consists of capillary water and adsorbed water. The physical mechanisms of water retention in porous media are reviewed first. A new water retention

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model is proposed, which takes into account (i) the difference between capillary water and adsorbed water and (ii) the possibility of capillary condensation. A simple shear strength criterion that only involves capillary water is derived based on the analysis of a partially saturated two-cylinder system and is justified against thermodynamic principles. The proposed model is compared with existing shear strength criteria, against experimental data in the literature.

2 Theoretical backgrounds

Based on the qualitative discussion of the distribution of water on the surface of two soil particles [21], [22] developed a quantitative unitary approach to capillarity and adsorption in porous media. The liquid-vapor interface is considered as a surface of either constant partial specific Gibbs free energy (Ψ) or constant chemical potential (μ) which consists of an adsorptive component and a capillary component. The capillary component (C) of the potential depends on the liquid-vapor interface mean curvature (κ), and the adsorptive component (A) depends on the normal distance of the interface from the solid (h). [20] modified the unitary approach of [22] by establishing a linkage between modern interface science concepts and equations for adsorption developed by [5] and [23]. A considerable simplification was also introduced by [20], by considering the absorptive and capillary contributions separately, which is termed the shifted Young-Laplace equation. The most important advantage of the shifted Young-Laplace approach is that this approach is able to separate adsorbed water from capillary water at a given chemical potential, which makes the derivation of the capillary water retention equation possible.

The total potential of water in soil is equivalent to the chemical potential of soil water at a chosen temperature and atmospheric pressure [24]. Given that water is incompressible, the negative total potential is equal to the total suction (ψ, $= u_a - u_w$, u_a being the pore air pressure and u_w the pore water pressure) divided by the water density (ρ). Total suction consists of matric suction and osmotic suction, where the matric suction represents the interaction of soil water with both the mineral soil skeleton and the gas phase, and the osmotic suction depends on the salt concentration in the bulk water [3]. For non-saline soils, it is reasonable to assume that the matric suction is much greater than the osmotic suction, and the total suction is equal to matric suction approximately.

The increase of adsorbed water film due to suction decrease can be explained by the absorptive term in shifted Young-Laplace equation, which states $h^3 \propto 1/\psi$. Theoretically, the adsorptive water film can increase to 7×10^{-8}m when the suction is equal to 0.01kPa [25], which is only valid for the simplest condition that no mutual attraction between two adjacent surfaces exists (see Figure 1Erreur ! Source du renvoi introuvable.a). The mutual attraction between two adjacent surfaces affects the stability of adsorbed liquid films. At a certain film thickness (h_c), two water films can form a liquid bridge,

followed by a spontaneous filling up of the gap between two adjacent surfaces (see Figure 1c). This phenomenon is known as the capillary condensation or the 'snap-off', which has been observed experimentally with the surface force apparatus by [26]. For a system consisting of two planar surfaces parallel with each other (see Figure 1Erreur ! Source du renvoi introuvable.b), [20] concluded that the critical film thickness (h_c) at the point of capillary condensation is approximately equal to one third of the distance between two surfaces (H). Therefore, the critical film thickness may be much smaller than the theoretical thickness due to the capillary condensation. When the thickness of water films is less than the critical film thickness, the liquid can stably be retained on the surface by adsorption. As shown in Figure 1b, along with decreasing suction (namely an imbibition process), the adsorbed water film thickness increases first and the quantity of adsorbed water increases correspondingly. Once the thickness of adsorbed water film reaches the critical film thickness, capillary condensation occurs (see Figure 1c) and the liquid retained in soil pores is governed by the capillarity only [20].
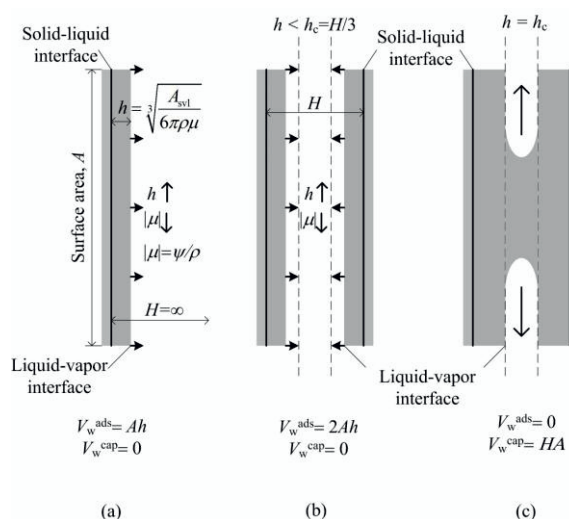

Figure 1 Increase of adsorbed liquid film and capillary condensation.

3 Water retention model for capillary and adsorbed waters

Pore space geometry is traditionally simplified as a 'bundle of cylindrical capillaries' (BCC), such as in [27]. However, the BCC model only takes into account the cylindrical capillarity and ignore the role of specific surface area and liquid films adsorbed on the surface. Recently, [20] suggested a new pore geometry model to facilitate the consideration of both adsorption and capillary processes. The new pore geometry model consists of a series of capillary pores (either angular or circular) to represent capillary-dominated processes and a series of slit-shaped spaces to represent adsorption-dominated phenomena. The adsorption process is governed by the adsorptive term of the shifted Young-Laplace equation and capillary process is governed by the

capillary term of the shifted Young-Laplace equation. The theoretical water retention curve considering both capillarity and adsorption was firstly proposed by [28]. Compared with other water retention models in the literature, the most important feature of the model by Or and Tuller is its ability to separate the individual contributions of absorptive and capillary processes from a given chemical potential and take into account capillary condensation as well. However, the equation of the theoretical water retention curve derived by [28] is very complicated and difficult for practical applications.

In addition to theoretical water retention models, some practical models were proposed to describe the capillary/adsorbed water retention behaviours based on the theoretical understandings. For example, [18] proposed a water retention model that retains the form of the widely-used empirical water retention equations, such as the Brooks-Corey model and the van Genuchten model, in the wet range and the form of an adsorption equation developed by [17] in the dry range. Similarly, [19] applied the adsorption equation developed by [17] to a water retention model developed by [29].

$$\theta = (\theta_s - \theta_a)C(\psi) + \theta_a, \quad C(\psi) = \frac{1}{2}\mathrm{erfc}\left(\frac{\ln(\psi/\psi_m)}{\sqrt{2}\zeta}\right), \quad (1)$$

where θ is the volumetric water content, θ_s the volumetric water content at fully saturated state, θ_a the volumetric water content ascribed to adsorption, erfc() the complementary error function, ψ_m the suction that corresponds to the median pore radius, and ζ the standard deviation of the log-transformed pore radius. According to [17], θ_a can be written as

$$\theta_a = \theta_0 A(\psi), \quad A(\psi) = \left(1 - \frac{\ln\psi}{\ln\psi_d}\right), \quad (2)$$

where θ_0 is the volumetric water content due to adsorption at a suction of 1kPa and ψ_d suction at oven dryness. Experimental results have shown that oven dryness generally corresponds to a finite suction of 10^6 kPa. Equation (2) was widely adopted in the literature [18, 19, 30]. However, equation (2) does not take into account capillary condensation. The neglect of capillary condensation results that the volumetric water content due to adsorption keeps increasing along with the decrease of suction, which conflicts with the experimental observation described in the previous section.

Sharing the theoretical concept delivered by [28] and practical method by [19], a simple equation of water retention curve is proposed here to consider capillarity and adsorption separately, which can be written as follows:

$$S_r = S_r^{cap} + S_r^{ads}, \quad (3)$$

where S_r is the degree of saturation, S_r^{cap} the capillary component of degree of saturation and S_r^{ads} the adsorptive component of degree of saturation. Following [19], the two-parameter equation proposed by [29] to quantify the

capillary component (i.e., capillary water retention curve, CWRC):

$$S_r^{cap} = \left(1 - S_r^{ads}\right)C(\psi), \quad (4)$$

The adsorptive component S_r^{ads} (i.e., adsorbed water retention curve, AWRC) can be described by the following equation:

$$S_r^{ads} = \frac{\theta_a}{\theta_s}(1 - P_{cc}), \quad (5)$$

where θ_a is the volumetric water content due to adsorption ignoring capillary condensation, which can be described by equation (2) and P_{cc} stands for the possibility of capillary condensation ($0 \leq P_{cc} \leq 1$). Obviously, $P_{cc} = 1$ if soil is fully saturated ($S_r = S_r^{cap} = 1$) and $P_{cc} = 0$ if capillary water is equal to zero ($S_r^{cap} = 0$). The simplest equation meets the above requirement of P_{cc} can be written as: $P_{cc} = S_r^{cap}$. Therefore, the adsorbed water retention curve can be specified as

$$S_r^{ads} = \alpha A(\psi)(1 - S_r^{cap}), \quad (6)$$

where α is a parameter ($=\theta_0/\theta_s$) that is related to the theoretical degree of saturation due to adsorption (without considering capillary condensation) when the suction is equal to 1kPa. Combining equations (3), (4) and (6) yields close-form equations for WRC, CWRC and AWRC:

$$\begin{cases} \mathrm{WRC}: & S_r = S_r^{cap} + S_r^{ads} = \dfrac{C(\psi) + \alpha A(\psi) - 2\alpha C(\psi)A(\psi)}{1 - \alpha C(\psi)A(\psi)} \\[2mm] \mathrm{CWRC}: & S_r^{cap} = \dfrac{C(\psi) - \alpha C(\psi)A(\psi)}{1 - \alpha C(\psi)A(\psi)} \\[2mm] \mathrm{AWRC}: & S_r^{ads} = \dfrac{\alpha A(\psi) - \alpha C(\psi)A(\psi)}{1 - \alpha C(\psi)A(\psi)} \end{cases}$$

$$(7)$$

where $C(\psi)$ and $A(\psi)$ can be found from equations (1) and (2), respectively.

Erreur ! Source du renvoi introuvable. shows a typical WRC, the corresponding CWRC and AWRC predicted by the proposed equations in the space of degree of saturation and suction. A similar plot can be found in [28]. As shown in Figure 1, the CWRC is a typical s-shape curve which indicates more capillary pores are dried along with increasing suction. The size of a drying pore is inversely proportional to the suction (or the negative chemical potential), which is quantitatively governed by the capillary term of the shifted Young-Laplace equation. The AWRC shows a peak value at an intermediate suction, which implies the degree of saturation due to adsorption decreases after an initial increase, with decreasing suction. This hill-shape curve can be explained by surface adsorption and capillary condensation jointly. At the beginning, the capillary water quantity is very limited at the dry end and thus the possibility of capillary condensation is very low. Therefore, along with decreasing suction, the adsorbed water film thickness increases freely and the degree of saturation based on adsorbed water also increases accordingly (see Figure 1b). However, the increase of

the adsorbed water film thickness leads to the increase of the possibility of capillary condensation, which will result in the decrease of adsorbed water quantity (see Figure 1c). When soil is fully saturated, adsorbed water is completely removed because of capillary condensation.

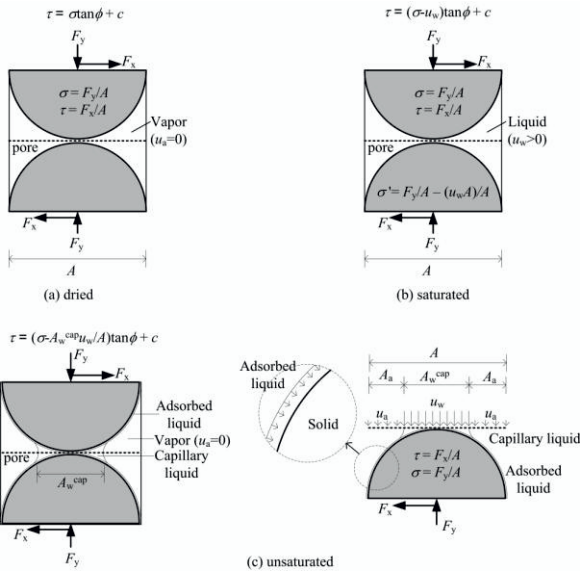# 4 Shear strength criterion for unsaturated soils

Figure 2 Contribution of pore liquid and vapor on the shear strength.

Soil is a frictional material and its shear strength can be attributed to the friction (ϕ) and the inherent cohesion (c) between soil particles. For example, for a system consisting of two dry cylinders in the atmosphere (see Figure 2a), the shear strength (τ) of this system is determined by the apparent contact stress (σ), for a given material (i.e., both ϕ and c are given). If the system is fully saturated with a liquid of a bulk liquid pressure u_w (Figure 2b), the true contact stress (σ') is equal to the apparent contact stress (σ) subtracts the liquid pressure (u_w) and the shear strength (τ) is determined by the effective contact stress (σ'). When the system is partially saturated, liquid-vapor interface is formed. The pore liquid retained by suction consists of capillary liquid that exists between two particles and adsorbed liquid that wraps the surface of each particle, according to the shifted Young-Laplace equation. If this unsaturated system is virtually separated at the potential slip plane, only the pressure of the capillary liquid can be exposed and affects the contact stress (σ') between two particles. The pressure of adsorbed water, which is caused by the van der Waal's forces between liquid molecules and solid molecules, is always treated as an internal stress and does not affect the contact stress (σ') between two particles. Therefore, only the pressure of capillary water affects the shear strength of this partially saturated system. In general, if neither u_a nor u_w is equal to zero, the shear strength can be written as

$$\tau = \left(\sigma - u_a + (u_a - u_w)\frac{A_w^{cap}}{A} \right) \tan \phi + c , \qquad (8)$$

where A_w^{cap}/A is the normalised area of capillary water. If the adsorption is ignored and only capillary liquid is taken into account, the above equation can be rewritten through replacing A_w^{cap} by A_w:

$$\tau = \left(\sigma - u_a + (u_a - u_w)\frac{A_w}{A} \right) \tan \phi + c , \qquad (9)$$

where A_w/A is the normalised area of water.

In the past several decades, adsorbed water was broadly ignored in unsaturated soil mechanics, equation (9) were widely adopted, such as in [2]; [3]; [31] and [12]. The normalised area of water (A_w/A) is assumed to be in direct proportion to the normalised volumetric water content (i.e., degree of saturation, S_r) in the soil by applying Greens theorem [2, 32]. Therefore, equation (9) can be written as

$$\tau = \left(\sigma - u_a + (u_a - u_w)S_r \right) \tan \phi + c , \qquad (10)$$

Compared with equation (9), equation (10) is more practical and can be directly applied to predict the shear strength of unsaturated soils. Equation (10) highlights the effect of hydraulic behaviour (water retention) on mechanical behaviour (shear strength) of unsaturated soils. In other words, the shear strength of an unsaturated soil is related directly to its water retention curve ($S_r - \psi$). Equation (10) is widely adopted in the literature. However, as shown in a competitive study on eight shear strength criteria [13], equation (10) usually much overestimates the shear strength of unsaturated soils, especially for clayey soils at a high suction level.

To achieve a better prediction to shear strength behaviour of unsaturated soils, a revision to equation (10) has been suggested through replacing the degree of saturation (S_r) by the effective degree of saturation (S_e) [2, 9, 14, 16, 33]. Pore water is divided into two categories: free water and residual water. Residual water is considered to be attached to the particles or trapped in the micropores. The intrinsic velocity of residual water is also equal to that of the soil solids, whereas the remaining water fraction, referred to as free water, is able to flow through the pores with distinct intrinsic velocity [7]. The volume of residual water is assumed to be a constant and cannot be removed by increasing suction. Only free water contributes to shear strength of unsaturated soils. By employing the effective degree of saturation, equation (10) can be revised as

$$\tau = \left(\sigma - u_a + (u_a - u_w)S_e \right) \tan \phi + c , \quad S_e = \frac{S_r - S_r^{res}}{1 - S_r^{res}} , \quad (11)$$

where S_e is the effective degree of saturation and S_r^{res} is the residual degree of saturation. Equation (11) implies that the shear strength of an unsaturated soil is related directly to its effective water retention curve ($S_e - \psi$), EWRC, other than the WRC. Equation (11) was widely used in unsaturated soil mechanics recently, such as in [33]; [14]; [9], [16] and [15]. Compared with equation (10), equation (11) gives a better prediction to the

observed shear strength of unsaturated soils [13]. However, for a clayey soil with a high suction, equation (11) still overestimates shear strength. In addition, dividing the pore water into a changeable free water and a constant residual water suffers from several shortcomings. Firstly, all the liquid retained in soil pores, including the water trapped in the micropores, can be fully removed if zero water content refers to oven dryness (namely, 10^6 kPa suction). Secondly, the quantity of water attached to soil particles (i.e. adsorbed water) can also change with the variation of suction and the occurrence of capillary condensation.

Equations (10) and (11) can be further revised by including an additional strength fitting parameter κ, such as in [1], [2] and [4] :

$$\tau = \left(\sigma - u_a + (u_a - u_w) S_r^k \right) \tan \phi + c \quad , \quad (12)$$
$$\text{or } = \left(\sigma - u_a + (u_a - u_w) S_e^k \right) \tan \phi + c$$

Compared with equations (10) and (11), equation (12) can provide a better prediction [13]. However, the determination of the additional parameter (k) in equation (12) requires more experimental data, which limits its application to practice. Another limitation is that this fitting parameter k is completely based on the regression of experimental results and has no clear physical meaning.

A fundamental but effective way to achieve a better prediction to the shear strength data is to acknowledge the difference between capillary water and adsorbed water. In analogy to the relationship between equations (9) and (10), a new shear strength criterion is proposed here based on equation (8), by assuming that the normalised area of capillary water (A_w^{cap}/A) is in direct proportion to the normalised volumetric capillary water content (i.e., degree of saturation due to capillary component, S_r^{cap}):

$$\tau = \left(\sigma - u_a + (u_a - u_w) S_r^{cap} \right) \tan \phi + c \quad , \quad (13)$$

Equation (13) implies that the shear strength of an unsaturated soil is related directly to its CWRC ($S_r^{cap} - \psi$) other than either WRC or EWRC. The capillary water retention curve can be determined by equation (7) with given water retention test results.

For triaxial conditions, the deviator stress (q) can be calculated via $q = M(p - u_a - p_s)$, with $M = 6\sin\phi/(3 - \sin\phi)$, p being the total mean stress, and p_s the apparent tensile strength that is equal to $-S_r(u_a - u_w)$, $-S_e(u_a - u_w)$ or $-S_r^{cap}(u_a - u_w)$, corresponding to equations (10), (11) or (13), respectively.

5 Validation and comparison

In this section, different shear strength criteria including equations (10), (11) and (13) are compared with each other against experimental data. The common ground of the above three equations is that: (1) all of them involve water retention equations (i.e., WRC / EWRC / CWRC) and (2) all of them do not include any additional fitting parameter that has no clear physical meaning. The value

of S_r involved in equation (10) and S_r^{cap} in equation (13) are calculated by the 3-parameter (ψ_m, ζ and α) water retention model developed in this paper (i.e., equation (7)). The value of S_e in equation (11) is calculated by 3-parameter (a, n and S_r^{res}) VG model developed by [11].
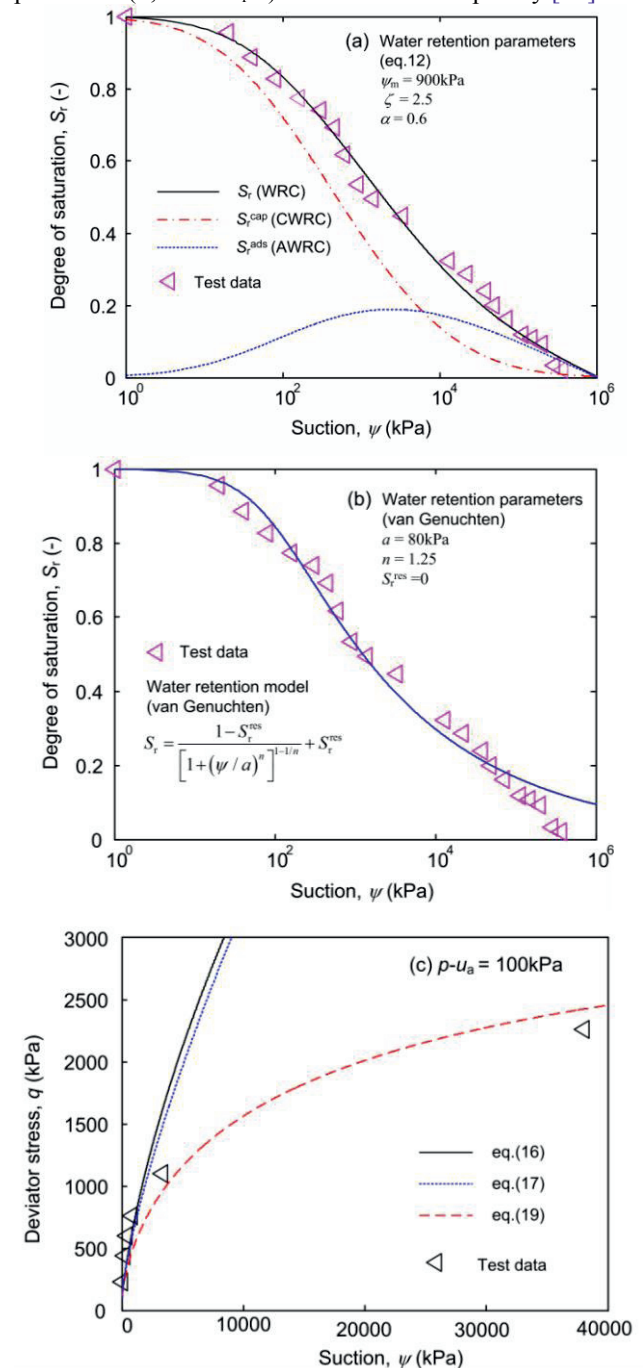

Figure 3 Measured water retention behaviour and shear strength for Nanyang expansive soil (data after Zhang et al., 2015) and model predictions.

Zhang et al. conducted a series of triaxial tests on Nanyang expansive soil under various suctions (0, 200, 400, 800, 3290, 38000kPa) [34]. Nanyang expansive soil is a weakly expansive soil with a liquid limit of 38.8% and a plasticity index of 21.6. X-ray diffraction analysis indicated that the predominant minerals of the soil were 62% quartz, 12% albite, 10% microcline, 9.9% illite and 5.8% montmorillonite, with a small percentage of

kaolinite (0.3%). The critical state parameter was measured by saturated triaxial tests ($M = 1.05$). The water retention behaviour of Nanyang expansive soil was measured by drying tests for almost the entire saturation range ($S_r = 1\sim0.02$). For the low suction range (0~1000kPa), the axis translation technique (ATT) was employed for measuring the water retention behaviour, whereas the vapor equilibrium technique (VET) was adopted to impose suction beyond 1000kPa. The suction of 3290 kPa was achieved by VET using K_2SO_4 and the suction of 38000kPa by VET using NaCl. The air entry value of this weakly expansive soil is about 15kPa. As shown in Figure 3a and 3b, the proposed model (equation(7)) and the VG model are used to simulate the observed water retention behaviour, respectively. For Nanyang expansive soil, the proposed water retention curve gives a better simulation to the experimental results than the VG model. The shear strength data of Nanyang expansive soil with different suctions (0~38000kPa), at a net confining pressure of 100kPa, are replotted in Figure 3c as well as the predictions from the three models. The comparison (see Figure3c) shows that the proposed model predicts the measured shear strength reasonably well, especially for high suctions (such as 3290kPa and 38000kPa).

6 Conclusion

Based on the theoretical analysis provided by Tuller et al. [20], this paper proposed a new water retention model which is able to separate the capillary water from the adsorbed water and consider capillary condensation. A new shear strength criterion for unsaturated soils, which involves the capillary part of the proposed water retention model, was then derived and validated.

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