Network from the circles of the same radius for sustainable energy-efficient roof structures

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Abstract. In this research paper, we investigated one of the methods of formation of geometric networks of arches of the same radius using regular spherical polyhedra. The variants of cutting sustainable energy-efficient coatings of buildings in the form of spherical domes are proposed. The task conditions of placing the specified network on the sphere are set. The criterion for evaluating the effectiveness of solving the problem is the minimum number of standard sizes of segments of the dome arches, the possibility of using pre-assembly technologies. The solution of one variant of the problem as placing the network on a spherical icosahedron and, accordingly, on a sphere is given. The placement of arches of one radius on the sphere, different from the location in the form of meridians, has an effective solution in the form of a network with minimal dimensions of arch segments and with nodes of paired arches comprised on the basis of circles of the same radii formed on the ground of regular spherical polyhedra. The problem is solved by constructing and combining in a system of regular spherical polyhedral with independent frameworks of arches of the same radius on the basis of paired circles of equal radius.

1 Introduction

In a geodesic dome, on the basis of a regular spherical polyhedron (icosahedron), there is a possibility of such placement in the form of a network with the minimum sizes of arch segments and with pair arches unites comprised on the basis of circles of the same radii formed on the ground of the correct spherical polyhedra, with preservation of the minimum number of standard sizes which will provide an effective arrangement of support unites much lower or higher than the equator and at one, quite certain level [1-13]. In the investigated regular spherical polyhedron (Fig. 1), the vertices of the planes are denoted as O, and the centers of the planes O_o and the radius of the edges of one radius as r.

If we analyze the possibility of such a variant of the geometric network on the sphere, we conclude that if we choose as the poles for the construction the centers of the planes O_o, the correct spherical twenty-triangle (icosahedron), and for the geometric network (figure 1), a simplified scheme of figure 2 should be shown, where the arcs r are cut off on the edges and

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bisectors of the faces of the arc x and the arc y, which determine the position of the main figures of the network. On the circles of the same radius, the arc z is cut off, the arc x and the arc y. New radius of the equator is shown for illustration purposes and is denoted as r_o.

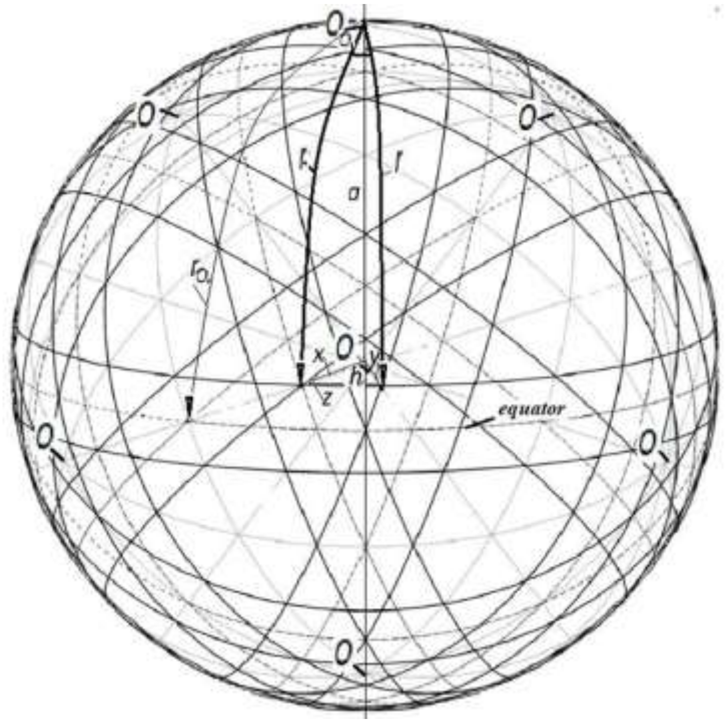

Fig. 1. The geometric network diagram of the same radius circles on the basis of the correct spherical icosahedron; O_o – pole in the center of the face of the regular icosahedron on the sphere; O - the vertices of the angles; r – circle radius of paired arches.

2 Decision 1

The diagrams in figures 1 and 2 show the placement of paired circles of the same radius [1-13] on the basis of a spherical icosahedron. On the known location conditions of the centers of the circles in the centers of opposite planes, the problem of forming a geometric network on a sphere with the centers of unites located on the circles parallel to the equator circle (i.e., at one level) is reduced to solving a system of equations for spherical triangles shown in figure 1. We tentatively identify the parameters of a spherical face $O\ O\ O$ of a regular twenty-triangle inscribed in a sphere as a spherical triangle. According to the problem, we take the interior angle at the Zenith considered the large triangle in figure1 $\mathrm{O_o} = 20°$, total arc z arc tightens triangles in the polar angles equal to x and y. Interior angles of triangles around the vertices of the faces of the icosahedron, the subtending arc$z$ is equal to 108^0 and,respectively, with the sides x and a is equal $O_{x\mathrm{a}} = 108^0$ and with sides a and y -$O_{y\mathrm{a}} = 144^0$. Using the known Napier's expressions [14] for the sides and angles of right-angled spherical triangles, we obtain

$$\cos r = \cos a \cos x + \sin a \sin x \cos 108^0.$$
$$\cos z\, r = \cos a \cos y + \sin a \sin y \cos 144^0. \quad (1)$$
$$sin 10^0\ sin\, r\ =\ sin\, 0{,}5z.$$

$$\frac{tg\,h}{tg\,x} = cos\,72^0.$$

$$\frac{tg\,h}{tg\,\,y} = cos\,\,36^0.$$

where r is the radius of a circle parallel to the equator; x, y, z are the arcs of a spherical triangle; h is the height of this triangle; 20° is the inner angle O_0 that pulls the arc z; a is the arc from the pole in the center of the face to the top of the face ($a = 79{,}18770479^0$) – all in the form of polar angles.

From equations (1) we obtain

$$\frac{cos\,\,72^0 tg\,x}{cos\,36^0} \quad = tg\,y. \tag{2}$$

Let us transform the system of equations (1 and 2) using the formulas [14-22]

$$sin\,x = \frac{2tg0{,}5x}{1 + tg^2 0{,}5x}\,; \tag{3}$$

$$cos\,x = \frac{1 - tg^2 0{,}5x}{1 + tg^2 0{,}5x}.$$

We use the same formulas for the parameter y.

Let $n^2 = tg^2 x$ and $m^2 = tg^2 y$ and $b = \frac{cos\,\,72^0}{cos\,36^0}$; then $m = bn$.

We continue to transform the system of equations (1 and 2), and so we obtain

$$cos\,a\frac{1 - m^2}{1 + m^2} + sin\,a\,\,\frac{2m}{1 + m^2} cos144^0 - \cos a\frac{1 - n^2}{1 + n^2} - sin\,a\,\,\frac{2n}{1 + n^2} cos\,108^0 = 0 \tag{4}$$

$$cos\,a\frac{1 - b^2 n^2}{1 + b^2 n^2} + cos144^0 sin\,a\,\,\frac{2b\,n}{1 + b^2 n^2} - \cos a\frac{1 - n^2}{1 + n^2} - sin\,a\frac{2n}{1 + n^2} cos\,108^0 = 0,$$

$$cos\,a(1 - b^2 n^2)\,(1 + n^2) + 2b\,cos144^0 sin\,a\,\,n\,(1 + n^2) - \\ - \cos a\,\,(1 + b^2 n^2)(1 - n^2) - 2\,cos\,108^0\,sin\,a\,n(1 + b^2 n^2) = 0,$$

$$(\,b^2 sin\,a - b^2\cos a)n^3 + 2(cos144^0 b\,sin\,a - cos\,108^0\,b^2\,sin\,\,a)n^2 \\ + 2(\cos a - b^2 \cos a)n \\ + 2(cos144^0\,\,b\,sin\,\,a - cos\,108^0\,s\,in\,\,a) = 0 \tag{5}$$

When the coefficients are substituted, equation (5) takes the form

$$0{,}1159385898n^3 - 0{,}51849288n^2 + 0{,}32044556n + 0{,}39732726 = 0.$$

From where, using [15] $n = tgx = 0{,}23855897$

Calculate $x = 13{,}4176398^0$..

From expression (2) we find

$$h = arctg\,cos\,72^0\,tg\,x = 4{,}21614835^0;$$

Considering that $\sin 10^0 \sin r = \sin 0{,}5z$; $z = 19{,}872796°$.

We find the value of r from the formula (1): r=83,5671836°.

3 Decision 2

Independent variants of arrangement of paired circles can be combined in pairs, or all three independent systems can be combined into a single frame of paired arches [1-13]. To do this, we estimate the possibility of forming the second variant of such geometric network on the sphere (figure 2), where the vertices of the faces *O*, the correct spherical twenty-triangle (icosahedron) should be chosen as the poles for the construction.

Let us show the construction of the circles on the figure 2, where r of the arc cut off on the edges and the bisectors of the faces of the arc x and arc y, which completely determine the position of the major figures of this network. With almost identical schemes for the solution of the second problem, only the value of the known parameter a – arc from the pole at the top of the face to the other vertex of the face will change - all in the form of polar angles. On the circles of the same radius, the z arc, which tightens the x and y arcs, is also cut off. The radius of the equator is shown for clarity and is denoted as r_O.

The scheme of the figure 2 shows the placement of paired circles of the same radius [1-9] on the basis of a spherical icosahedron. Under condition of the known location of the centers of the circles in the centers of opposite faces, the problem of forming a geometric network on a sphere with the centers of nodes located on the circles parallel to the equator circle (ie, at one point), will be reduced to solving a system of equations for spherical triangles, shown in figure 1.

We predetermine the parameters of spherical triangles formed on a spherical face $O\ O\ O$ of a regular icosahedrons. To solve our problem by means of spherical trigonometry, we can determine the parameters of arch arcs cut off by large circles forming a regular polyhedron. Then according to the problem, we need to define a new received interior angle at the Zenith considered the large triangle in figure 2, the total arc z arc also tightens triangles in the polar angles equal to x and y, h is the height of this triangle.

Using the known Napier's expressions [14] for the sides and angles of right-angled spherical triangles, respectively, we obtain again a system of equations (1). Ie, in the end, we can use the unchanged equations (1-5) with partial replacement of the interior angles of the sectors that intercept arcs of the arched elements $a = 63{,}434966^0$.

$$\cos r = \cos a \cos x + a \sin x \cos 108^0.$$

Taking into account the substitutions of the formula (4)

$$\cos r = \cos a \frac{1-n^2}{1+n^2} \cos x + \sin a \frac{2n}{1+n^2} \cos 108^0; \tag{6}$$

The resulting equation takes the same form as (5). Taking into account that the values of x and y, h vary with the new value of a, the simplified equation of the known radius is

$$(\cos r + cos\ a)n^2 + + + 2\ cos\ 108^0\ s\ in\ a\ n + \cos r - sin\ a = 0\ . \tag{7}$$

The radius value remains unchanged at $r =$ 83,567183607°.

Then tg $x = -1{,}77611201671$; $x = 29{,}3807097°$;

$h = 9{,}869813208°$.

Finding the inner angle centered on the pole O_x, which draws the arc x, which is equal to

$$O_x = 22{,}081339°.$$

Similarly, we find the value of y.

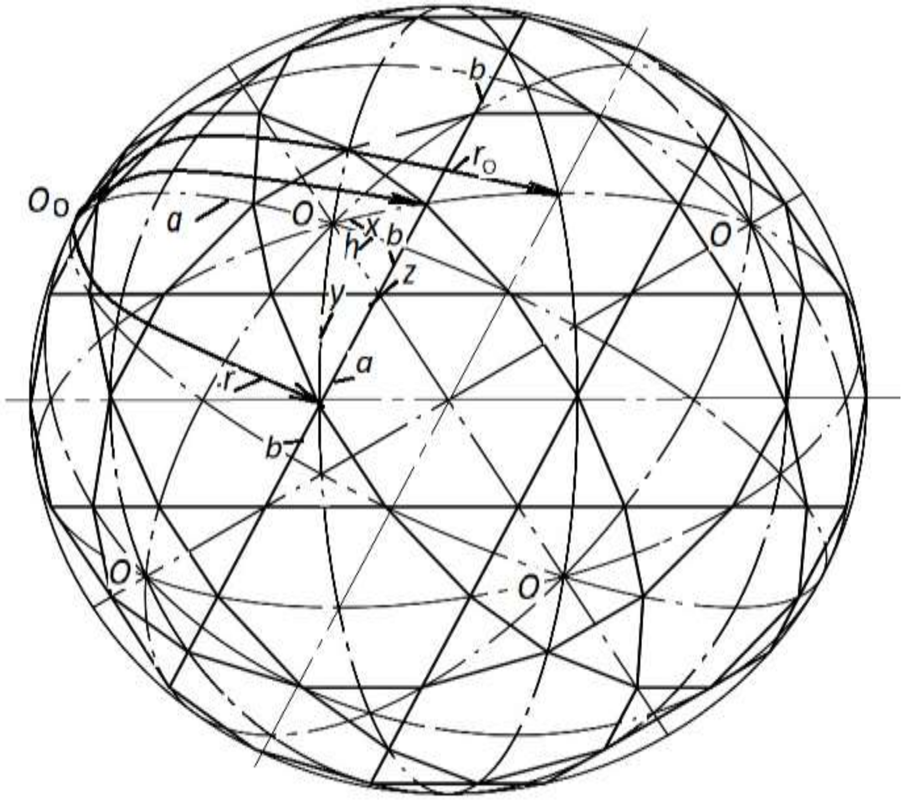

Fig. 2. Spherical plane parameters with vertices $O\ O\ O$ of the correct spherical icosahedron; O_o– pole in the center of the face of the regular icosahedron on the sphere; $O,\ O,$ – vertices of edges; $x, y,\ z$ – arches of the triangle, h - its altitude ; $r_o,\ r$ - the radii of the circles in the polar angles.

The scheme of figure 4 shows the placement of circles of the same radius, which allows to create an effective design of the dome of two independent systems of frames in the form of the same arches [14-21].

The chaotic arrangement of the arches of two combined variants of the schemes in figure 4 actually contains two regular simple framework of paired arches.

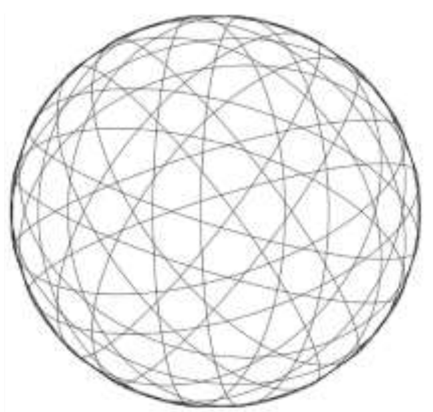

Fig. 3. Geometric network diagram of circles of the same radius on two separate arched frames on the basis of the correct spherical icosahedron.

4 Summary

The proposed cutting solution is the geometric basis of the geodesic dome formed by paired circles of the same radius. It allows us to greatly simplify the solutions of all the shell unites in which no more than four elements converge; including the support unites, since the base is made with nodes at the same level. These sustainable energy-efficient structures can be assembled from individual arches, and the technology of integration assemblies of the dome elements can also be used.

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