Evaluation of the coefficient of earth pressure at rest (K_0) of a saturated-unsaturated colluvium soil

Lorena Gomes Abrantes[1,*] and *Tácio Mauro Pereira* de Campos[1]

[1]Pontifical Catholic University of Rio de Janeiro, Department of Civil and Environmental Engineering, Brazil

Abstract. To predict the stress-strain behaviour of soils under loading it is relevant the knowledge of its natural stress state, expressed by the coefficient of earth pressure at rest (K_0). There are correlations in the literature for K_0 determination that comes from researches developed considering sedimentary soils, typically from temperate or cold regions. In dealing with residual and colluvium soils, typical of tropical regions, it is not appropriate to use these correlations, since K_0 is affected by factors such as degree of weathering, laterization processes and suction, among others that also affect saturated sedimentary soils. This work analysed samples of a colluvium soil from a natural slope at the Pontifical Catholic University of Rio de Janeiro. For the determination of the coefficient of earth pressure at rest of the unsaturated colluvium soil, a flexible wall equipment, with a system of suction control was used. The influence of the net stress level on the value of the coefficient of earth pressure at rest was investigated under a constant suction of 10 kPa, following a loading and unloading cycle of applied vertical stress. The coefficient of earth pressure at rest of the saturated colluvium (null suction) was also determined using a stress-path, Bishop and Wesley type, servo-controlled triaxial equipment. The obtained results are presented and discussed.

1 Introduction

The necessity to predict the stress-strain behaviour of soils under loading makes it important to know the in situ stress state that occurs initially in the soil (Vaughan and Kwan [1]). The coefficient of earth pressure at rest is defined as the ratio between the horizontal and vertical effective stresses. Thus, it is a parameter that expresses the natural state acting on the soil. Determining such value correctly should, therefore, be a common practice in engineering.

Although the earth pressure theory is commonly used, few types of studies have been carried out to evaluate the influence of suction and other factors on the coefficient of earth pressure at rest of unsaturated residual and colluvium soils. Recently, Alvim and de Campos [2] evaluated the influence of suction and weathering degree on the K_0 value of a residual soil. Nhut and Adel [3] carried out tests for the determination of K_0 of a collapsible soil formed by kaolin and sand.

To evaluate the influence of the suction and of the net stress level on the coefficient of earth pressure at rest of the colluvium soil, laboratory tests were performed. An equipment with control of suction, similar to a triaxial equipment, developed at the Geotechnical and Environmental Laboratory of the Pontifical Catholic University of Rio de Janeiro (PUC-Rio) was employed for that. To obtain the coefficient of earth pressure at rest on the same soil in the saturated condition, CK_0 tests were performed on a servo controlled triaxial equipment developed at Imperial College, London.

The soil samples used in this work were retrieved from a natural slope located at the Experimental Campus II of the Pontifical Catholic University of Rio de Janeiro, in Rio de Janeiro, Brazil.

2 The coefficient of earth pressure at rest

Following the well-known Bishop's effective stress definition for unsaturated soils, given by:

$$\sigma' = (\sigma - u_a) - \chi(u_a - u_w) \quad (1)$$

where σ' and σ are, respectively, effective and total normal stresses, u_a is air pressure, u_w is water pressure and χ is the Bishop's parameter for effective stress, the following formulation is derived for the coefficient of earth pressure at rest, K_0 (Bishop [4]):

$$K_0 = \frac{\sigma'_h}{\sigma'_v} = \frac{(\sigma_v - u_a) - \chi(u_a - u_w)}{(\sigma_h - u_a) - \chi(u_a - u_w)} \quad (2)$$

where the sub-indexes h and v correspond, respectively to horizontal and vertical stresses.

To include the behaviour of over-consolidated soils in the K_0 formulation, Andrawes and El-Sohby [5] defined such parameter as (Equation 3): the ratio between the

*Corresponding author: lorenagomesabrantes@hotmail.com

increase of the effective minor principal stress (horizontal effective stress) and the increase of the effective major principal stress (vertical effective stress), with the restriction of the deformation in the direction of the minor principal stresses.

$$K_0 = \frac{\Delta\sigma'_h}{\Delta\sigma'_v} \tag{3}$$

Taking into account the Bishop's effective stress equation, and assuming the soil as being homogeneous and linear elastic, Lu and Likos [6] expanded the Hoke's law for unsaturated soils as indicated in Equations 4 to 6.

$$\varepsilon_x = \frac{\sigma_x - u_a}{E} - \frac{\nu}{E}\left(\sigma_y + \sigma_x - 2u_a\right) + \frac{(1-2\nu)\chi(u_a - u_w)}{E} \tag{4}$$

$$\varepsilon_y = \frac{\sigma_y - u_a}{E} - \frac{\nu}{E}\left(\sigma_x + \sigma_z - 2u_a\right) + \frac{(1-2\nu)\chi(u_a - u_w)}{E} \tag{5}$$

$$\varepsilon_z = \frac{\sigma_z - u_a}{E} - \frac{\nu}{E}\left(\sigma_x + \sigma_y - 2u_a\right) + \frac{(1-2\nu)\chi(u_a - u_w)}{E} \tag{6}$$

where:
ε_x, ε_y, ε_z = strains in directions x, y and z;
σ_z = vertical total stress;
σ_x e σ_y = horizontal total stresses;
ν = Poisson's ratio;
E = elasticity modulus;

To satisfy the condition of null lateral deformation, the following equation applies:

$$\varepsilon_x = \varepsilon_y = 0 \tag{7}$$

With that, it is possible to arrive at the following relationship:

$$\frac{\sigma_h - u_a}{\sigma_v - u_a} = K_0 = \frac{\nu}{1-\nu} - \frac{1-2\nu}{1-\nu}\,\frac{\chi\,(u_a - u_w)}{\sigma_v - u_a} \tag{8}$$

Equation 8 was that proposed by Lu and Likos [6] to define the coefficient of earth pressure at rest in unsaturated soils. This equation indicates that, theoretically, K_0 would tend to reduce with an increase in the matric suction.

Following a different approach, which does not include any definition of effective stresses, Fredlund and Rahardjo [7] proposed the following equation to define K_0 in unsaturated materials:

$$K_0 = \frac{(\sigma_h - u_a)}{(\sigma_v - u_a)} \tag{9}$$

Fredlund and Rahardjo [7] do not say anything on the effect of the air pressure on the K_0 value. However, if the values of the horizontal and vertical stresses are kept constant, an increase in the air value would imply in a decrease of the K_0 value as defined by Equation 9.

In the present work, K_0 values were computed using both Equations 2 and 9. To use Bishop's approach (Eq. 2) the value of χ was computed following the proposal by Khalili e Khabbaz [8] as indicated in Equation 10:

$$\begin{cases} \chi = 1 \text{ for } S < S_e \\ \chi = \left(\frac{S}{S_e}\right)^r, S \geq S_e \end{cases} \tag{10}$$

where:
S = current matric suction (equal to $u_a - u_w$);
S_e = suction at the air entry point;
r = material dependent parameter, adopted in this work as -0,55.

3 Devices and testing techniques

According to Ting et al [9], the laboratory methods for determining the coefficient of earth pressure at rest include flexible wall and rigid wall types of test.

Flexible wall tests are those in which a rubber membrane confines the sample, such as in triaxial tests in which compression of the sample occurs while lateral deformation is completely impeded (Bishop [10]). Flexible wall tests have the advantage of the absence of friction between the soil and its confining wall, and the possibility of controlling the applied lateral and axial stresses while lateral deformation is kept null through monitoring radial deformations. In addition, measurement of lateral stresses is possible if an incompressible confining cell fluid and a rigid triaxial cell are used (e.g Ting et al [9], Bishop and Henkel [11]. Disadvantages are related to keep a radial strain null throughout the sample height when testing control is based on radial strain measurements (Ting et al. [9]).

Rigid wall tests comprise those in which a rigid lateral border confines the sample, such as the case of oedometric tests. In these tests, the requirement of null lateral deformation is typically obeyed. As pointed out by Ting et al [9], the main disadvantage of such tests is on the measurement of the horizontal stress due to the occurrence of lateral friction between the sample and the rigid confining wall of the oedometer ring.

Regardless the type of device (with flexible or rigid wall) a further testing complexity is related to the need of either to measure the suction within the unsaturated sample being tested or to control such suction.

In the present work, two equipment, both comprising flexible wall devices, were used.

For the determination of the coefficient of earth pressure at rest in the saturated colluvium soil it was used a servo controlled triaxial equipment, developed at Imperial College (IC), London. It comprises a stress path controlled triaxial cell, type Bishop & Wesley (see Figure 1), with an internal load cell for measuring and control axial (deviator) stress and an external displacement transducer for measuring axial displacements. Pressure transducers were used to measure and control cell and pore-pressures. An IC volume change device was employed to measure water volume changes of the 1,5" diameter x 3.0" height samples.

Fig. 1. View of the Bishop-Wesley triaxial cell

Stress path testing was controlled through high-resolution air pressure valves geared by step motors coupled to fine resolution gear boxes. The software TRIAX, developed by Toll [12], was employed to monitor all instrumentation and for full K_0 testing control.

The following procedures were adopted in the development of the K_0 tests in the triaxial equipment:

i) Saturation using the back pressure technique, with increments of chamber pressure in steps of 10 kPa per hour up to reaching a Skempton B parameter of 0.95;
ii) Isotropic consolidation to 30 kPa, in order to avoid an eventual failure of the sample by touching accidently the failure envelope in the very early stages of anisotropic consolidation. In this and all subsequent steps, the back pressure was kept constant and equal to 150 kPa, with drainage occurring both at the top and base of the triaxial specimen;
iii) Constant mean stress consolidation up to a deviator stress failing in a K_0 path estimated by using the well-known Jaky's Equation – $K_0 = 1\text{-sin } \phi'$. This step was introduced in order to speed up the subsequent anisotropic consolidation main testing step.
iv) Anisotropic consolidation following the K_0 stress path up to a vertical effective stress of 200 kPa.

Through the continuous measurement of the water volume variation of the saturated sample, the volumetric deformation of the specimen was defined. The condition of zero radial deformation was set by making the volumetric deformation equal to the monitored axial deformation. With such control criteria, the K_0 path was followed applying increments of vertical stress of 10 kPa/h. As the sample tended to expand radially, the axial stress increment would be halted and the cell pressure would be increased, also at a rate of 10 kPa/h, until returning to the condition of zero radial strain. In the case of radial contraction, the axial stress would be incremented while keeping the cell pressure constant.

For the determination of the coefficient of earth pressure at rest under unsaturated conditions it was used a special cell developed at PUC-Rio by Daylac [13]. It comprises a suction controlled rigid triaxial chamber with internal diameter of 4.5", with the triaxial sample having a diameter of 4" and height of 30 mm. A self-supporting loading frame was employed for axial loading of the sample (see Figure 2). Deaired water was used as confining fluid and an internal load cell, with a piston inside the chamber of the same diameter of the sample, was used to monitor vertical stresses. Owing to that, as the specimen is consolidated, its volume variation is equal to the volume occupied by the advancement of the piston. Thus, the lateral deformation of the specimen was kept to zero. Suction was controlled using the axis translation technique. The water pressure was applied at the bottom of the specimen, through a saturated high air entry value porous stone of 500 bars. Air pressure was applied at the top of the sample, through a dry, coarse porous stone. Figure 3 shows a general view of the equipment set up, without the inclusion of the self-supporting loading frame.

Horizontal stress variation under increments of vertical stress and net stress was measured by a pressure transducer with 0.5 kPa resolution, inserted at mid-high of the rigid triaxial chamber. Corrections for fluid, tubing and chamber variations of volume due to pressure variations were applied to the measured horizontal stress through adequate preliminary calibrations.

Fig. 2. View of the suction controlled K_0 cell with the self-supporting loading frame

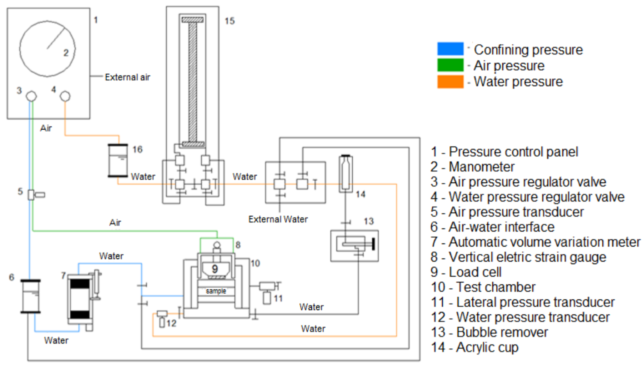

Fig. 3. General set up of the unsaturated K_0 equipment

The following procedure was adopted in the development of the unsaturated K_0 tests:

i) Sample set up at its natural water content;
ii) Application of initial suction and monitoring of equilibrium of sample volume change through measurements of water entering or leaving the sample under zero axial loading;
iii) Application of axial loading and unloading, in steps, and monitoring of sample volume change and of axial displacement changes until achieving equilibrium in each step. After equilibrium, measurement of the lateral pressure.

4 Material characteristics

Figure 4 and Table 1 show results of characterization tests performed on the tested material. According to the USCS classification system, the colluvium soil is a clay of high plasticity (CH). Indeed, it comprises a gap-graded soil, with a clay fraction of 60.1% and a coarse fraction of 33.0%.

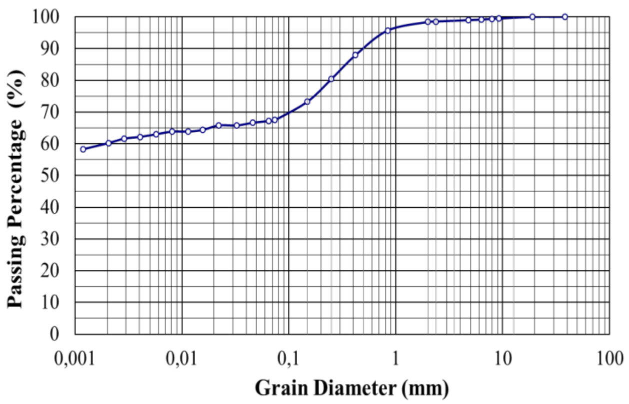

Fig. 4. Grain size distribution curve of the colluvium

Table 1. Results of characterization tests

Grain size (%)				Atterberg limits (%)		
Gravel	Sand	Silt	Clay	w_L	w_P	PI
1.6	31.4	6.9	60.1	65.9	28.6	37.2

Results of chemical analysis indicated that the colluvium presents a high degree of laterization. Mineralogical analysis showed that the main mineral present in the fine fraction is kaolinite, with traces of goethite. In the coarse fraction prevails quartz, with traces of grenade and magnetite.

Table 2 shows average values of physical indexes of undisturbed samples of the colluvium.

Table 2. Mean physical indexes

w_{nat} (%)	G_s	γ_{nat} (kN/m³)	γ_d (kN/m³)	e	n (%)	S (%)
25.0	2.79	17.4	13.8	0.97	49.9	69.9

4.1 Soil water retention curve

The soil water retention curve (SWRC) was obtained using the filter paper technique (e.g. Marinho [14]), following both drying and wetting paths.

Drying path was followed after saturation of sub-samples by capillarity. Wetting path was followed after air drying of sub-samples (refer to Abrantes [15] for methodology details). Whatman 42 filter papers were used in all cases.

To obtain the SWRCs shown in Figure 5, the filter paper calibration equations proposed by Chandler and Gutierrez [16] were employed. The data points above 1 kPa of suction shown in this figure represent average results from filter papers put in contact with both top and bottom of each sub-sample. The data shown at the matric suction of 1 kPa represent, indeed, a condition of full soil saturation. Thus, the volumetric water content at such suction equals the average porosity of the initially saturated and dried sub-samples.

As indicated in Figure 5, the colluvium soil presents a double structure type of SWRC. Hysteresis occurs under wetting and drying conditions.

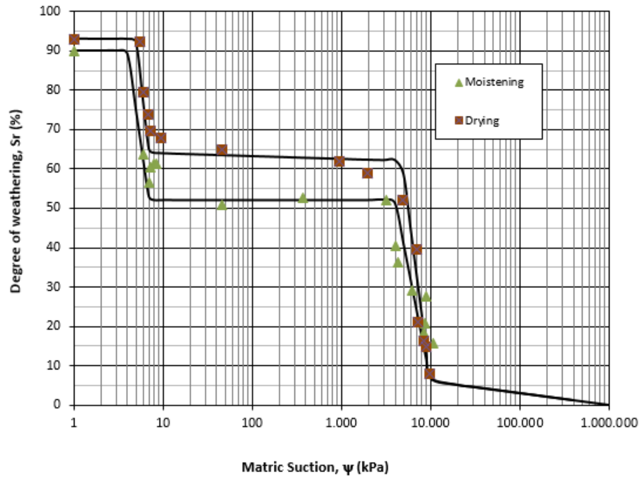

Fig. 5. Soil water retention curves

The curves fitting the data points in Figure 5 were obtained according with the following equation, proposed by Gitirana and Fredlund [17]:

$$S=\frac{S_1-S_2}{1+\left(\psi/\sqrt{\psi b1.\psi res1}\right)^{d1}}+\frac{S_2-S_3}{1+\left(\psi/\sqrt{\psi b2.\psi res1}\right)^{d2}}+\frac{S_3-S_4}{1+\left(\psi/\sqrt{\psi b2.\psi res2}\right)^{d3}}+S_4 \quad (11)$$

where Ψ_{b1} and Ψ_{b2} = values of suction at the first and second points of air entry; Ψ_{res1} and Ψ_{res2} = values of suction at the first and second points of residual saturation; $S_1 = 1$; $S_2 = S_{res1}$; $S_3 = Sb$; $S_4 = S_{res2}$ and $d_j = 2\exp[1/\ln(\psi_{j+1}{}^{a} / \psi_j{}^{a})]$ = weight factors, j=1,2,3.

Table 3 shows the fitting parameters of the SWRCs following Gitirana and Fredlund [17].

Table 3. Fitting parameters of the SWRCs

Parameter	Drying Curve	Wetting Curve
Ψ_{b1}	5	4
ψ_{res1}	7	7
S_{res1}	0.64	0.52
Ψ_{b2}	4,800	4,000
S_b	0.62	0.52
ψ_{res2}	10,000	10,000
a	0.01	0.01

5 Results

The results of the investigations performed are presented and discussed considering, first, those associated to the saturated colluvium and, then, the unsaturated material.

5.1 Saturated Colluvium

Considering exclusively the anisotropic consolidation step of the CK_0 triaxial test performed, Figure 6 shows that the condition of zero radial strain has been achieved. Thus, a K_0 path was followed. As indicated in Figure 7, the obtained K_0 value for the saturated colluvium soil was equal to 0.36.

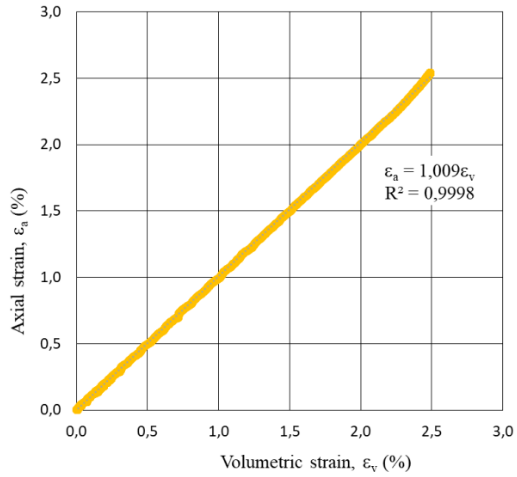

Fig. 6. Relationship between volumetric and axial strains in the anisotropic consolidation of the saturated triaxial test

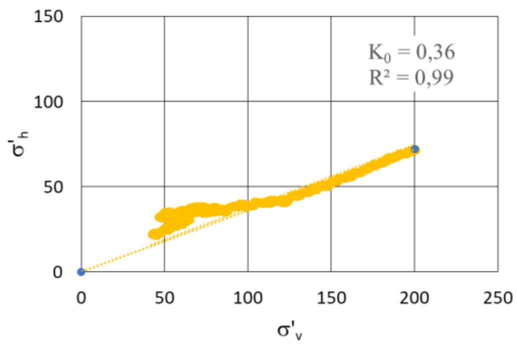

Fig. 7. K_0 evaluation for the saturated colluvium soil

Such K_0 value differs from that obtained by Delcourt [18], which found a K_0 of 0.43 when testing a saturated colluvium from another site in Rio de Janeiro.

5.2 Unsaturated Colluvium

Table 4 and Figure 8 show the values of the coefficient of earth pressure at rest, K_0, obtained following both loading (increase of total vertical stress) and unloading (decrease of total vertical stress) paths. The shown K_0 values were computed using both Equation 9 (named Fredlund in the figure and table) and Equation 2 (named Bishop in the figure and table).

Table 4. Test results with 10 kPa suction.

Fredlund			Bishop		
σ_v-u_a	σ_h-u_a	K_0	σ'_v	σ'_h	K_0
144,7	39,7	0,27	189,1	87,8	0,44
371,0	61,2	0,17	444,6	134,9	0,30
634,1	125,0	0,20	699,5	190,3	0,27
883,8	182,9	0,21	907,6	206,7	0,23
703,7	195,1	0,28	739,0	234,4	0,32
454,8	139,7	0,31	493,2	178,1	0,36
164,2	68,4	0,42	191,5	95,7	0,50

In the horizontal axis of Figure 8 are plotted values of the mean stresses p and p' defined as:

$$p = \frac{\sigma_v + 2\sigma_h}{3} - u_a \qquad (12)$$

$$p' = (\sigma'_v + 2\sigma'_h)/3 \qquad (13)$$

As mentioned at the end of item 2, to use Bishop's approach (Eq. 2) the value of χ was computed following the proposal by Khalili e Khabbaz [8]. In the present case, for the applied suction of 10 kPa and considering the SWRC shown in Fig. 5, a value of χ of 0,68 was adopted considering both the loading and unloading paths followed in the tests.

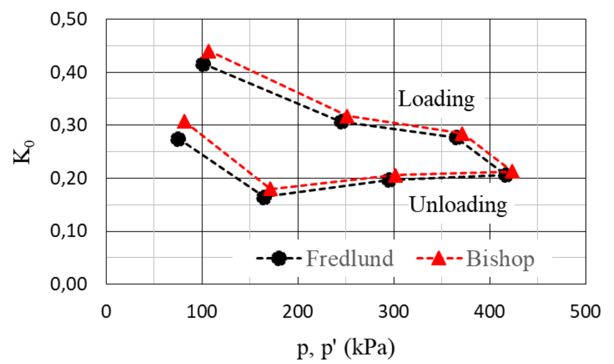

Fig. 8. Variation of K_0 with mean stresses

It can be seen in Figure 8 that the K_0 value decreases with the increase of the mean effective stress, either when defined in terms of net stress (Fredlund) or effective stress (Bishop). In addition, it can be seen that lower values of K_0 are obtained as the axial stress is decreased (unloading). Furthermore, for the low suction level employed (10 kPa) little difference was found for the values of K_0 computed as proposed by Fredlund and Rahardjo [7] and in terms of effective stresses as defined by Bishop [2].

Considering data available in the literature, Alvim and de Campos [2] report a K_0 variation between 0.94 and 0.51, under soil loading at a constant suction of 10 kPa, in an unsaturated residual mature soil. Such type of soil (mature residual soil) is considered, at PUC-Rio, as similar to a colluvium soil. Thus, as the K_0 in the present work varied from 0.17 to 0.44 under loading, it appears that the origin / structural characteristics of the colluvium / mature soil may interfere in the laboratory-determined value of K_0.

6 Conclusion

The laboratory tests for the determination of the coefficient of earth pressure at rest (K_0) of the colluvium soil showed that, under unsaturated conditions, K_0 varies with both net stresses and effective stresses. For the low constant suction herein considered (10 kPa) variations of K_0 obtained using both net stresses and effective stresses are negligible.

The value of K_0 under unsaturated conditions varied during loading and unloading, being higher under unloading.

Under saturated condition, it was found a value of K_0 of 0.36 for the tested colluvium soil. Such value is higher than the average value of K_0, equal to 0,26, obtained under a suction of 10 kPa, following a loading path. This is in agreement with what has been theoretically advanced in the literature, which indicates that the K_0 value decreases with an increase of suction or decrease of the degree of saturation.

Further studies however are required to support the above findings. In particular, effects of suction and of structure of tropical soils on the K_0 value are to be investigated.

This work was developed within a PRONEX Project supported by the Brazilian (CNPq) and the Rio de Janeiro State (FAPERJ) Research Councils.

References

1. P.R. Vaughan, C.W. Kwan. Weathering, structure and in situ stress in residual soils. Géotechnique, **34**, 1 43-59 (1984).
2. T.S. Alvim, T.M.P. de Campos. Study of the coefficient of earth pressure at rest for unsaturated residual soils with different weathering degrees. *2nd Panamerican Conference on Unsaturated Soils - PanAm-UNSAT*, Texas, USA (2017).
3. C.E. Torres, J.E. Colmenares. Influence of Confining Stress on the Small Strain Stiffness of a Residual Soil under K_0 Conditions. *2nd Panamerican Conference on Unsaturated Soils - PanAm-UNSAT*, Texas, USA (2017).
4. A.W. Bishop. The principle of effective stress. Tecknik Ukebland, **106**, 39, 859-863 (1959).
5. K.Z. Andrawes, M.A. El-Sohby. Factors Affecting Coefficient of Earth Pressure Ko. Journal of Soil Mechanics and Foundation Engineering Division, Proceedings ASCE, **99**, SM7, 527-539 (1973).
6. N. Lu, W.J. Likos. *Unsaturated Soil Mechanics*. John Wiley & Sons, (2004).
7. D.G. Fredlund, H Rahardjo. *Soil Mechanics for Unsaturated Soils*. Ed: John Wiley e Sons. United States of America, New York, 517 (1993).
8. N. Khalili, M.H. Khabbaz. A unique relationship for the determination of the shear strength of unsaturated soils. Géotechnique, **48,** 5, 681-687 (1998).
9. C.M.R Ting, G.C. Sills, D.C. Wijeyesekera. Development of K0 in soft soils. Géotechnique, **44**, 1, 101-109 (1994).
10. A.W. Bishop. Test requirements for measuring the coefficient of earth pressure at rest. Brussels Conference on Earth Pressure Problems, Brussels, Belgium, **1**, 2-14, Discussion, **3**, 36-39 (1958).
11. A.W. Bishop, Henkel. The Measurement of Soil Properties in the Triaxial Tests. Edward Arnold (Publishers) Ltd., London, (1957).
12. D.G. Toll. Triax – User Manual. Geotechnical Systems Group, School of Engineering, University of Durham. DH1 3LE UK, version 4.2, (2002).
13. R. Daylac. Development and use of a Ko measuring cell with suction control. *Master's dissertation*, DEC/PUC-Rio, Rio de Janeiro, 180 (in Portuguese) (1994).
14. F.A.M. Marinho. Measurement of suction with the filter paper method. X Brazilian Congress of Soil Mechanics and Foundations Engineering, Foz do Iguaçu, PR, 2, 515-522, (in Portuguese) (1994).
15. L.G. Abrantes. Evaluation of the coefficient of earth pressure at rest (K_0) of a saturated-unsaturated colluvium soil. *Master's dissertation*, DEC/PUC-Rio, Rio de Janeiro, 150, (in Portuguese) (2019).
16. R.J. Chandler, C.I. Gutierrez. The filter-paper method of suction measurement. Geotechnique, **36**, 2, 265-268 (1986).
17. G.F.N. Gitirana Jr, D.G. Fredlund. Soil-water characteristic curve equation with independent parameters. Journal of Geotechnical and Geoenvironmental Engineering, **130**, 2, 209-212 (2004)
18. R.T. Delcourt. Effects of the weathering degree on the physical, chemical, mineralogical, microstructural and strength characteristics of a residual soil of phacoidal gneiss. *Doctoral thesis*, DEC/PUC-Rio, Rio de Janeiro, 286 (in Portuguese) (2018).