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Effect of External Magnetic Field on Dynamics of Two-dimensional Isotropic Conducting Flow

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Abstract. In this article, the impact of external uniform magnetic field on the dynamic characteristics and mixing parameters of two-dimensional isotropic magnetohydrodynamic (MHD) flow is investigated. For this purpose, the direct numerical simulation (DNS) is applied to two-dimensional incompressible Navier-Stokes and magnetic induction equations by pseudo-spectral method. Governing equations are considered in the N-S vorticity equations to guarantee the incompressibility conditions and remove the pressure term from equations. The Results of the calculations show that the deformation of vortexes by external magnetic field reduces the mixing efficiency. It is also demonstrated that in MHD flow the energy is exchanged by Lorentz force between the flow and the magnetic field in such a way that the kinetic energy decreases and consequently mixing of the fluid is reduced. This energy transfer causes reduction of viscous dissipation of energy and mixing efficiency, despite increasing the total dissipated energy rate.

1 Introduction

Magnetic fields influence many natural and man-made flows. They are routinely used in industry to heat, pump, stir and levitate liquid metals. There are also many natural phenomena affected by the magnetic field. Although there is no such thing as two-dimensional turbulence, and indeed it is true that all real flows are three dimensional, certain aspects of certain flows can be considered 'almost' two-dimensional. For example, the depth of the troposphere is very low compared to the atmosphere. Therefore the third dimension can be ignored if the large-scale investigation is desired. Also, in the laboratory or some industry, a strong magnetic field or intense rotation tends to suppress one component of motion. [1]

The investigation on two-dimensional (2D) magnetohydrodynamic (MHD) flow shows that 2D MHD turbulence is not as singular as 3D hydrodynamic (HD) turbulence and it has a less excited small-scale structure. But, it is more singular than 2D HD turbulence.[2, 3] Nonetheless, behavior of quasi-2D MHD flow is as 2D HD flow for moderate interaction parameter, and two-dimensional three-component type for high one. [4]

Study of kinetic and magnetic energy spectrum to explore energy transfer between different wavenumber shells demonstrates that there is a forward cascade of magnetic energy, an inverse cascade of kinetic energy, a flux of energy from the kinetic to the magnetic field, and a reverse flux which transfers the energy back to the kinetic from the magnetic. Energy transfer occurs due to two competing processes and there is critical magnetic to mechanical forcing amplitude ratio that can change inverse cascade of energy to forward cascade. However, the net transfer of energy is from kinetic to magnetic. [5-7] In reference [8] mode to mode energy transfer, energy cascade rates and shell to shell energy transfer in MHD turbulence is analytically computed. Also, in reference [9] it is shown that interaction parameter can effect on energy cascade slopes.

In order to investigate the influence of an external magnetic field on homogeneous MHD turbulence, study of the scaling behavior of longitudinal and transverse structure functions in homogeneous and isotropic MHD shows that unlike HD flows, there are no substantial differences between longitudinal and transverse structure functions in MHD turbulence. [10] In [11], it is shown that anisotropy is developed through the combined effects of an externally imposed constant magnetic field and viscous and resistive dissipation at high wavenumbers; its strength is characterized by the initial interaction parameter, the ratio of the large eddy-turnover time to the Joule-dissipation time. In addition, in reference [12] it is shown that no long-time anisotropy is to be expected without the presence of dissipation, and the smaller dissipation coefficients, the greater the degree of anisotropy is likely to be. In other research, decay of a weak large scale magnetic field in 2D MHD flow and dependence of the turbulent diffusivity on the magnetic Reynolds number and the energy of the large-scale magnetic field is investigated. [13]

Another important characteristic in turbulent MHD flow which is routinely applied in industry such as pump and stir liquid metals is mixing process. In many studies, mixing process is investigated experimentally or by numerical of some passive scalars, while less numerical research has been done on analysis of mixing characteristics in MHD flows. [14-16]

In this article the effect of uniform and constant external magnetic field in mixing properties of isotropic MHD flow is investigated by analyzing large and small scale eddies dynamics; In order to explore mutual interaction of velocity and magnetic field, MHD flow with moderate magnetic Reynolds number is considered and solved by use of DNS. The results of this research can be useful in phenomenology and also prediction of mixing behavior of some industrial equipment as well as natural phenomena.

For this, governing equations, initial conditions and numerical method are considered in section 2. In section 3, the accuracy of the code is verified and in section 4, the results and their analysis are presented.

2 Material and methods

2.1 Governing equations

The evolution of incompressible MHD turbulence can be determined by solving the governing equations of MHD which are a combination of a reduced form of Maxwell's equations and the Navier-Stokes (N-S) equation. In this case we use the N-S equation in the form of the vorticity in order to eliminate the pressure variable. [17]

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times \left(\mathbf{U} \times \boldsymbol{\omega} - \mathbf{b} \times \mathbf{J}\right) + \upsilon \nabla^2 \boldsymbol{\omega} \quad (1)$$

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times \left(\mathbf{U} \times \mathbf{b}\right) + \lambda \nabla^2 \mathbf{b} \quad (2)$$

$$\nabla \cdot \mathbf{U} = 0 \quad (3)$$

$$\nabla \cdot \mathbf{b} = 0 \quad (4)$$

where

$$\mathbf{b} = \frac{\mathbf{B}}{\sqrt{\rho \mu_0}} \quad (5)$$

$$\boldsymbol{\omega} = \nabla \times \mathbf{U} \quad (6)$$

$$\mathbf{J} = \nabla \times \mathbf{b} \quad (7)$$

$$\lambda = \left(\mu \sigma\right)^{-1} \quad (8)$$

where $\mathbf{U}$ is the incompressible velocity field, $\boldsymbol{\omega}$ is the vorticity field, $\mathbf{B}$ is magnetic field, $\mathbf{J}$ is current density, υ is constant kinematic viscosity and λ is magnetic diffusivity coefficient. Furthermore, In Eq. (1), (2) and (4), we use Alfv´en units ($\mathbf{b}$), i.e., the magnetic field is scaled to the Alfv´en velocity and measured in the same units as the bulk velocity.

As it is shown, there is the mutual interaction of a magnetic field and fluid which arises from their relative movements. Relative movements of the magnetic field and the fluid causes an induced magnetic field and an induced current that are expressed by the first R.H.S of Eq. (2); this leads to changes in total magnetic field such that the fluid appears to 'drag' the magnetic field lines along with it. In contrast, the combined magnetic field interacts with the induced current density to give rise to a Lorentz force per unit volume which is shown as second term of R.H.S of Eq. (1). This acts on the fluid to oppose the relative motion. [18] These mutual interactions lead to anisotropy and also some changes in geometric and dynamic fluid properties effect on mixing characteristics.

In addition, the parameters presented in Eq. (9) to (14), are important to identify the characteristics of MHD flow.

$$l = \sqrt{\frac{E_\upsilon}{Z}} \quad (9)$$

$$\tau = \frac{l}{\sqrt{E_\upsilon}} \quad (10)$$

$$\mathrm{Re} = \frac{Ul}{\upsilon} \quad (11)$$

$$\mathrm{Re}_m = \frac{Ul}{\lambda} \quad (12)$$

$$\mathrm{Pr}_m = \frac{\upsilon}{\lambda} \quad (13)$$

$$N = \frac{b^2 l}{\lambda U} \quad (14)$$

In Eq. (12), relative magnitudes of induced and dissipated magnetic field are measured. If Re_m is moderate or high, there is a strong two-way interaction between velocity and magnetic field, and oscillatory behavior is expected. At the other extreme, when Re_m is small, the imposed magnetic field is barely perturbed by the induced currents and the relative motion between the imposed field and the conductor is suppressed as the induced currents convert kinetic energy into heat.

2.2 Flow field configuration

In this paper, the influence of an external magnetic field on two-dimensional homogeneous isotropic MHD turbulence is investigated. For this purpose MHD equations are solved with periodic boundary conditions in a two-dimensional box and the size of the computational domain is 2π in each direction. In addition, initial conditions of homogeneous isotropic vorticity field is generated in Fourier space as Eq. (15)

$$\hat{\omega}(k,0) = E(k,0) e^{i 2\pi\theta} \quad (15)$$

$$i = \sqrt{-1} \quad (16)$$

That θ is a random phase between 0 and 2π with uniform probability distribution, and $E(k,0)$ is an initial energy spectrum which is demonstrated in Eq. (17).[19]

$$E(k,0) = \frac{Q}{k_p}\left(\frac{k}{k_p}\right)^7 \exp\left[-3.5\left(\frac{k}{k_p}\right)^2\right] \quad (17)$$

k_p is the peak wave number which, in two-dimensional turbulence, is related to the length scale.

$$k_p = \sqrt{\frac{7}{8}}\, l^{-1} \quad (18)$$

The reconstruction of homogeneous isotropic vorticity field is followed by a freely decay development run until a mature spectrum with physical phase relation among different Fourier modes is established. Since during this development run the integral length scale increases, to restore the desired length scale, the peak wave number k_p in the initial energy spectrum (17) is set as $0.9l$.

Afterwards, Fourier amplitudes of the time-evolved isotropic field are rescaled in order to match the prescribed initial turbulent kinetic energy E_υ. [19]

In addition, initial magnetic field is considered external constant and uniform at moderate Reynolds number in direction of +y. In this state, there are both mutual interaction between velocity and magnetic field and also suppression of total energy due to conversion of kinetic energy into heat by induced currents.

2.3 Numerical method

In order to identify imposed magnetic force effects on mixing characteristics, the geometric and dynamics of homogenous isotropic turbulence (HIT) conducting flow is investigated by direct numerical simulation (DNS). Because of periodic boundary condition of flow, the pseudospectral method is used. In this method, using Fourier transforms, Navier Stokes nonlinear equations are converted to ordinary equations. In order to reduce the calculation time, nonlinear terms are calculated in the physical space and then transmitted into Fourier space that causes aliasing errors. In this research, aliasing errors are removed using the 2⁄3-rule.
These equations can be easily solved in Fourier space by using any conventional numerical time integrating methods, such as fourth-order Runge–Kutta scheme which is implemented in this research. [20, 21]

3 Code validation

The DNS code is developed on the basis of section 2 formulas and used for Orszag-Tang MHD initial conditions. Energy dissipation rate (EDR) is obtained from Eq. (19) and compared with [3] (see Figure 1) that conform properly.

$$\varepsilon(t) = \upsilon \iint \omega^2 dxdy + \lambda \iint J^2 dxdy \qquad (19)$$

4 Result and discussion

In this study, the same initial HIT (see Figure 2) with different magnitude of magnetic fields are solved with a resolution of 2048^2 Fourier modes (as shown in Table 1). In each case the flow is evolved up to 30 times of τ.

As shown in Figure 3, in the absence of a magnetic field, the vorticity field remains isotropic; however its length scale grows due to the reverse energy cascade.

Table 1. The specification of different initial conditions

b_{ext}	Pr_m	Re_m	Re	υ	l
0	-	6.2	62	0.005	0.027
1	0.1	6.2	62	0.005	0.027
2	0.1	6.2	62	0.005	0.027
3	0.1	6.2	62	0.005	0.027

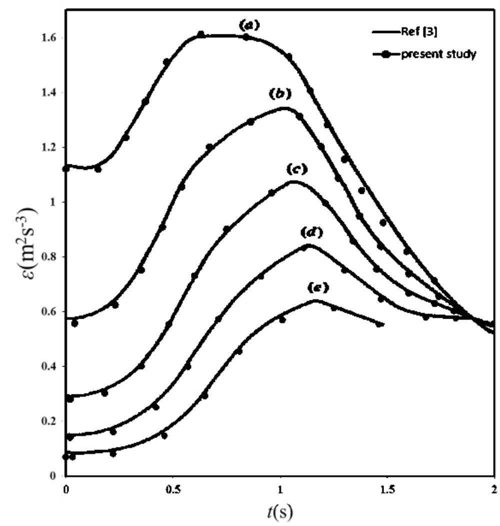

Fig. 1. Conformity of Energy dissipation rate with reference [3]. (a) $\upsilon = \lambda = 0.08$, (b) $\upsilon = \lambda = 0.04$, (c) $\upsilon = \lambda = 0.02$, (d) $\upsilon = \lambda = 0.01$, (e) $\upsilon = \lambda = 0.005$

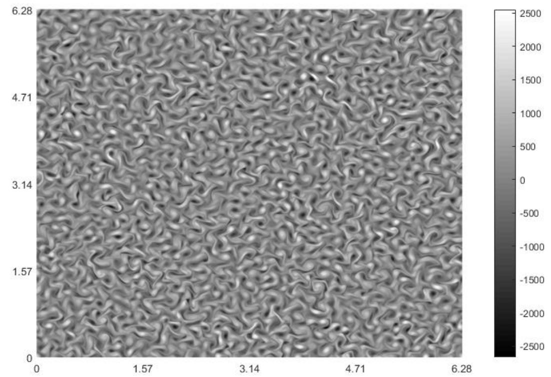

Fig. 2. Isotropic vorticity field in (τ=0)

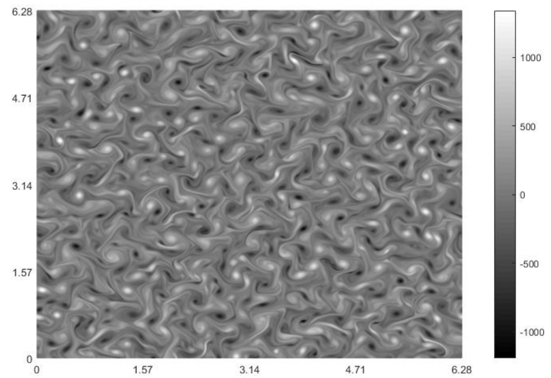

Fig. 3. Evolution of vorticity field after 30 τ without magnetic

In contrast, figure 4 shows that the magnetic field acts to shape the turbulence by extruding eddies along the magnetic field lines which leads to development of anisotropy in the flow field; hence some changes in structure and topology of the vorticity field are occurred. This trend is increased by enhance of magnetic field.

These changes influence on many important flow properties, including mixing characteristics.

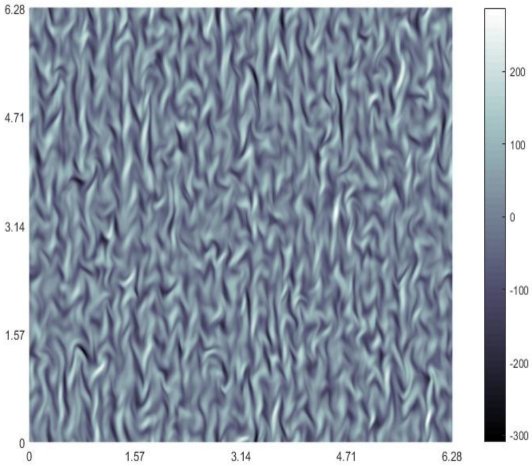

Fig. 4. Evolution of vorticity field after 30 τ with external magnetic field (b_{ext}=3)

Mixing efficiency is one of the most important parameters to identify the mixing characteristics. This parameter is related to stretching and folding of vorticity fronts with different vorticity values known as the enstrophy cascade that is the result of the non-linear interaction that produces and controls the strain field. This in turn gives rise to gradual erosion and filamentation of eddies and thus generates high vorticity gradients which are a generic feature of 2D turbulent dynamics.

We assess the efficiency of mixing as regards production of vorticity gradient. It can be verified that normalized instantaneous production rate of vorticity gradient obtained from Eq. (20) and called mixing efficiency. [22]

$$e = \frac{\sqrt{2}}{2}\cos(2\alpha) \tag{20}$$

Where α is the angle between the vorticity gradient and the compressing eigenvector d_2.

$$d_2 = \begin{bmatrix} -s_{12} \\ \sqrt{s_{11}^2 + s_{12}^2} + s_{11} \end{bmatrix} \tag{21}$$

$$s_{11} = \frac{2\partial u}{\partial x} \tag{22}$$

$$s_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{23}$$

Results illustrate that the intensification of magnetic fields and also the reduction of magnetic diffusivity cause the decrease of mixing efficiency (see Figure 5). This indicates that external magnetic field affect the direction of misalignment of vorticity gradient and the compressing eigenvector because of extruding eddies along the magnetic field lines.

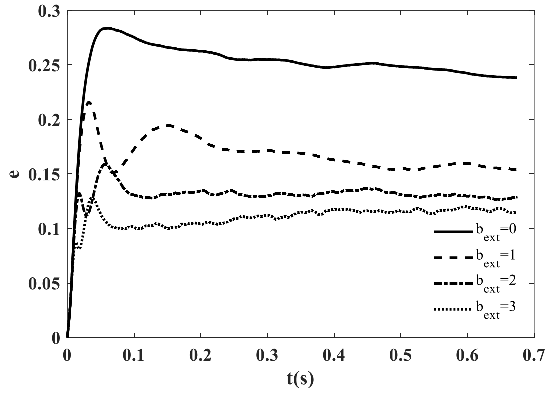

Fig. 5. Mixing efficiency changes of flow with external magnetic field (b_{ext}=0, 1, 2, 3)

In order to investigate the effective factors on reducing the mixing efficiency in the presence of imposed constant magnetic field, the dynamic characteristics affecting the mixing process should be studied. Since both large and small scale eddies affect the mixing properties via energy transfer and molecular diffusion, the changes of them due to magnetic field are analysed. [23]

One of the factors indicates the small scale dynamics and molecular diffusion, is EDR given by Eq. (19)

In two-dimensional hydrodynamic, turbulence energy dissipation is small and decreases during the time due to reverse energy cascade; whereas in two-dimensional MHD flows, EDR was found to increase strongly (see Figure 6). By decomposing the total EDR to viscous and magnetic EDR, it can be found that the main portion of the total EDR belongs to magnetic EDR; also viscous EDR of MHD is less than hydrodynamics'. Therefore the molecular diffusion and mixing by small scale are reduced.

In the other spectrum, in order to explore mixing characteristic related to large scale vortexes, energy transmission between fluid and magnetic field shows that in the presence of magnetic field, kinetic energy converts to magnetic energy; since magnetic EDR is much more than viscous EDR, this transfer causes more reduction of total and kinetic energy compared with the magnetic field absence.

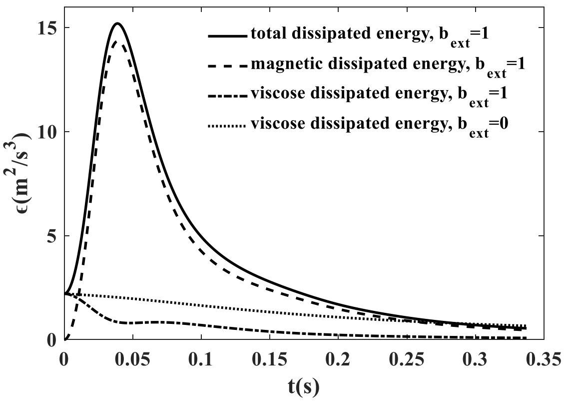

Fig. 6. Total, viscose & magnetic dissipation energy rate

To more explain, kinetic and magnetic energy rate are obtained from N-S and induced equations as Eq. (24) and (25).

$$\frac{DE_{\upsilon}}{Dt} = \mathbf{U}\cdot\left(-\nabla P + \upsilon\nabla^{2}\mathbf{U}\right) + \mathbf{U}\cdot(\mathbf{J}\times\mathbf{b}) \tag{24}$$

$$\frac{\partial E_{m}}{\partial t} = \nabla\cdot\left[\left(\mathbf{U}\times\mathbf{b}\right)\times\mathbf{b}\right] + \lambda\mathbf{b}\cdot\nabla^{2}\mathbf{b} - \mathbf{U}\cdot\left(\mathbf{J}\times\mathbf{b}\right) \tag{25}$$

In both above equations, the term $\mathbf{U}\cdot\left(\mathbf{J}\times\mathbf{b}\right)$ is appeared with opposite sign. This term including Lorentz force is responsible for transfer of kinetic to magnetic energy due to resistance of relative fluid and magnetic field motion and kinetic energy is reduced. Kinetic energy reduction leads to mixing of large scale eddies is decreased.

5 Conclusions

According to obtained results, it can be concluded that in the presence of constant and uniform external magnetic field, eddies are extruded along the magnetic field lines and some changes in vorticity structure and dynamics are occurred. These changes cause the reduction of mixing efficiency of flow. In order to detect the parameters effect on mixing decrease, dynamic characteristics of small and large scale are investigated. It is shown that in large scale vortexs, kinetic energy converts to magnetic energy because of the Lorentz force; therefore energy convection in large scale eddies is decreased. On the other hand, in the static magnetic field, magnetic EDR is much more than viscous EDR that means most of the total energy is converted to heat and the molecular diffusion is less than magnetic absence. The results of this fundamental research is useful to recognize phenomenology of some astrophysical properties as well as prediction of some metallurgical behaviors. These results indicate that in the case of a two-dimensional flow, constant magnetic field cause the decrease of mixing process in both large and small scales. In addition, increase of magnetic force leads to more suppression of kinetic energy in MHD flows.

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