Numerical Modeling of pipes conveying gas-liquid two-phase flow

Bakhtiyar Khudayarov*, *Kholidakhon* Komilova and *Fozilzhon* Turaev

Department of Higher Mathematics, Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Tashkent, 100000, Uzbekistan

Abstract. Results of studies of the oscillations of pipelines conveying a two-phase slug flow are presented in the paper. A viscoelastic model of the theory of beams and the Winkler base model are used in the study of pipeline oscillations with a gas-containing slug flowing inside. The Boltzmann-Volterra hereditary theory of the viscoelasticity is used to describe the viscoelastic properties of the pipeline material and earth bases. The effect of gas and liquid phases flow rates, influence of tensile forces in the longitudinal direction of the pipeline, parameters of Winkler bases, parameters of singularity in the heredity kernels and geometric parameters of the pipeline on the oscillations of structures with viscoelastic properties are numerically studied. It is revealed that an increase in the length of the gas bubble zone leads to a decrease in the amplitude and oscillation frequency of the pipeline. The critical rates for a two-phase slug flow are determined. It is revealed that an increase in the soil density of the bases leads to an increase in the critical rate of gas flow. It is shown that an account of viscoelastic properties of structure material and earth bases leads to a decrease in the critical flow rate.

1 Introduction

At present, pipeline transport is of great importance for the economic development of many countries all over the world. The fluid-conveying pipelines are the structural elements of many engineering structures. Pipelines are used in oil and gas facilities, chemical plants, gas processing plants, nuclear power plants and so on. Pipeline transportation differs from other types of transportation in its efficiency, convenience and continuity of delivery to the designated project. However, accidental pipeline breaking can damage the environment and pose a risk to human life. Oscillations of individual sections of pipelines conveying fluid are a difficult problem to study. To date, many dynamic models have been developed for solving such problems. Basically, these models describe the stages of the processes in a pipeline conveying fluid and gas. A significant number of publications are devoted to solving linear and nonlinear problems of oscillations and dynamic stability of pipelines [1-8].

Two-phase slug flows in pipelines occur in various processes in nuclear, oil and gas industry. Pipeline transportation of gas-containing fluid is accompanied by vibration effect on the pipeline, which, in some cases, leads to a rapid destruction of pipes. Accumulation

*Corresponding author: bakht-flpo@yandex.ru

and non-uniform distribution of gas along the length of the pipeline lead to pulsating vibrations, and to the displacement of the center of gravity of the flow along the pipe cross section, as a result, the pipeline receives an additional dynamic load.

A review of the literature that reflects the most up-to-date research progress in the field of oscillations caused by two-phase slug flow in pipelines is given in detail in [9]. In [10], the dynamics of pipelines conveying gas-containing two-phase slug flows is analytically and numerically analyzed. Parametric studies have been carried out to analyze the influence of the volume fraction of gas and volume flow on the dynamics of pipes conveying a two-phase air-water flow. In [11], experiments have been carried out in horizontal air-water pipes with a diameter of 32 and 50 mm. The results of experiments are compared with the theory presented in the paper, as well as with the hydrodynamic models previously published.

Currently, agriculture, oil and gas industry, and housing and communal services often face the problems in repairing, reconstructing, and restoring of pipelines due to the impact of various external factors. One of the ways to solve this problem is the use of modern, resource-saving, environmentally friendly technologies, which include the use of non-metallic, in particular, polymer composite materials [12,13]. Therefore, the methods and problems of the theory of hereditary elasticity attract much attention of researchers. There are a significant number of publications devoted to solving problems of calculating the characteristics of viscoelastic pipelines [14-17].

From the above review, we can conclude that the development of adequate models for the problem of oscillation of a viscoelastic pipeline conveying two-phase slug flow which take into account the work of the viscoelastic earth base, is a rather complex and relevant research task, which is the main objective of this study.

This paper is devoted to solving the above problems and its subject-matter is very relevant.

2 Problem formulation

Consider a viscoelastic pipeline in the form of a straight single-span beam hinged at both ends, lying on a viscoelastic base, described by the Winkler model. Choose a rectangular coordinate system so that the *x*-axis passes through the centers of gravity of the pipe sections in the supports with corresponding coordinates $x=0$ and $x=L$. The displacements of the points of the pipeline axis along the *y*-axis represent an unknown function of the deflections $w(x,t)$. The flow rate along the pipeline axis is U. Longitudinal oscillations of the pipeline are not taken into consideration. It is assumed that the motion is plane and the tube is nominally horizontal. The cross-sectional area of the flow is considered to be constant.

A pipeline conveying the gas-liquid two-phase slug flow is shown in Figure1. In the pipeline, several consecutive sections of slug units can be observed. Fig. 2 shows the gas bubble zone, the length of which is L_1, and the liquid slug zone, the length of which is L_2. The length of the pipeline is L ($L=L_1+L_2$).

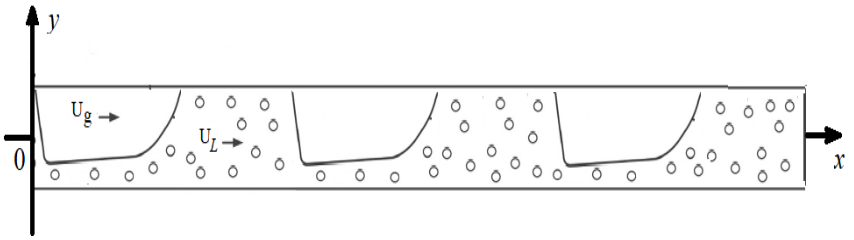

Fig.1. Diagram of a pipeline conveying gas-containing two-phase slug flow

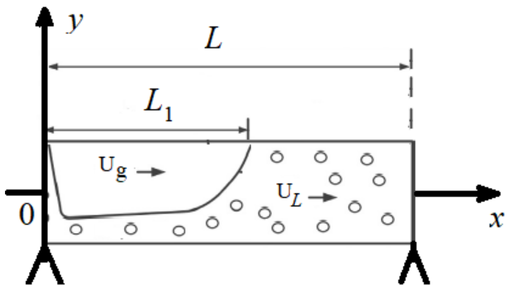

Fig.2. Diagram of a stable slug unit.

Based on [18], the equation of motion of the pipelines conveying a two-phase slug flow, considering the viscosity properties of structure and base material has the form:

$$EI\left(1-R^*\right)\frac{\partial^4 w}{\partial x^4}+2\left(m_L U_L+m_g U_g\right)\frac{\partial^2 w}{\partial t\partial x}+$$
$$\left(m_L U_L^2+m_g U_g^2\right)\frac{\partial^2 w}{\partial x^2}+(m_L+m_g+m_p)\frac{\partial^2 w}{\partial t^2}+ \qquad (1)$$
$$k_1\left(1-R_1^*\right)w-\left[N_0+\frac{E(1-R^*)A_0}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2}=0.$$

Here E is the modulus of elasticity of material; I is the moment of inertia of the pipeline section; EI is the bending stiffness of the pipe; w is the pipeline deflection; L is the length of the pipe between the supports; x is an independent variable, the longitudinal axial coordinate of the pipe; $w(x$, t) is the deflection in the section x at the point in time t; m_L, m_g and m_p are the masses of fluid, gas and pipe, respectively, related to the unit length of the pipeline; A_0 is the cross-sectional area of the pipe; U_L, U_g are the fluid and gas flow rates; k_1 is the bed coefficient of a viscoelastic base; N_0 is the compressive (tensile) force; R^*, R_1^* are the integral operators of the form: $R^*\varphi(t)=\int_0^t R(t-\tau)\varphi(\tau)d\tau$; $R_1^*\varphi(t)=\int_0^t R_1(t-\tau)\varphi(\tau)d\tau$; $R(t-\tau)$ and $R_1(t-\tau)$ is the Koltunov-Rzhanitsin relaxation kernel:

$$R\left(t-\tau\right)=A\cdot\exp\left(-\beta(t-\tau)\right)\cdot(t-\tau)^{\alpha-1}, \qquad (2)$$
$$R_1\left(t-\tau\right)=A_1\cdot\exp\left(-\beta_1(t-\tau)\right)\cdot(t-\tau)^{\alpha_1-1},$$
$$A>0,\quad \beta>0,\quad 0<\alpha<1,\ A_1>0,\quad \beta_1>0,\quad 0<\alpha_1<1;$$

t is the observation time; τ is the time point preceding the time of observation; A, A_1 are the viscosity parameters; β, β_1 are the attenuation parameters; α, α_1 are the singularity parameters determined by experiment.

Equation (1) is solved under the following boundary conditions

$$w(x,t)=\frac{\partial^2 w(x,t)}{\partial x^2}=0 \text{ at } x=0,\ x=L; \tag{3}$$

And initial conditions

$$w(x,0)=\vartheta(x), \quad \dot{w}(x,0)=\psi(x), \tag{4}$$

where $\vartheta(x)$, $\psi(x)$ are the given, smooth enough, functions in the field of arguments change.

3 Discretization and method of solution

Approximate solution of equation (1) is sought in the form:

$$w(x,t)=\sum_{n=1}^{N} w_n(t)\varphi_n(x) \tag{5}$$

where $w_n(t)$ are some functions to be defined, and functions $\varphi_n(x)$ are selected so that each term of the sum (5) satisfies the boundary conditions. In the case of a pipe hinged at the edges in the Bubnov-Galerkin method expansion (5), the approximating functions of the deflection are chosen in the form

$$\varphi_n(x)=\sin\frac{n\pi x}{L}\cdot \tag{6}$$

Substitute the function (5) into equation (1) and apply the Bubnov-Galerkin procedure to the latter. In the process of integration of equation (1) from 0 to L, flow parameters, including mass per unit length and flow rate for the gas and liquid phases located in the gas bubble zone and the liquid slug zone, are integrated separately in the interval from zero to L_1, and from L_1 to L (Figure 2). After simple transformations, a system of integro-differential equations for the coefficients (5) is obtained.

Introducing the following dimensionless values

$$\frac{x}{L},\ \frac{w}{L},\ \frac{t}{L^2}\sqrt{\frac{EI}{m_L+m_g+m_p}},$$

and maintaining the same notation, a system of integro-differential equations is obtained relative to w_n:

$$\sum_{n=1}^{N}\Delta_{kn}\ddot{w}_n+2\sum_{n=1}^{N}\left(\gamma_{Lkn}\beta_L u_L+\gamma_{gkn}\beta_g u_g\right)\dot{w}_n-$$

$$\sum_{n=1}^{N}\alpha_{0n}\left(\delta_{Lkn}u_L^2+\delta_{gkn}u_g^2\right)w_k+\alpha_{0n}\overline{N}_0 w_k+k_w\left(1-R_1^*\right)w_k+ \tag{7}$$

$$\gamma_1\alpha_{0k}\sum_{n,i=1}^{N}\phi_{ni}w_k(1-R^*)w_n w_i+\alpha_{0k}^2(1-R^*)w_k=0.$$

$$w_n(0)=w_{0nm};\ \dot{w}_n(0)=\dot{w}_{0nm};\ k=1,2,\dots,N.$$

Here $\Delta_{kn} = \delta_{Lkn} + \delta_{gkn}$; $\delta_{Lkn} = \int_0^{\overline{L}_1} \varphi_n(x)\varphi_k(x)dx$; $\delta_{gkn} = \int_{\overline{L}_1}^1 \varphi_n(x)\varphi_k(x)dx$;

$\gamma_{Lkn} = \frac{1}{n\pi}\int_0^{\overline{L}_1} \varphi'_n(x)\varphi_k(x)dx$; $\gamma_{gkn} = \frac{1}{n\pi}\int_{\overline{L}_1}^1 \varphi'_n(x)\varphi_k(x)dx$ - are the dimensionless coefficients; $\gamma_1 = \frac{A_0 L^2}{I}$; $\beta_L = \sqrt{\frac{m_L}{m_L + m_g + m_p}}$; $\alpha_{0n} = n^2\pi^2$; $k_w = \frac{k_1 L^4}{EI}$;

$\beta_g = \sqrt{\frac{m_g}{m_L + m_g + m_p}}$; $u_L = LU_L\sqrt{\frac{m_L}{EI}}$; $u_g = LU_g\sqrt{\frac{m_g}{EI}}$; $\overline{L}_1 = \frac{L_1}{L}$;

$\delta_n = \begin{cases} 0, & \text{если } n \neq 0, \\ 1, & \text{если } n = 0. \end{cases}$ $\overline{N}_0 = \frac{L^2 N_o}{EI}$; $\phi_{ni} = ni\pi^2(\delta_{n+i} + \delta_{n-i})/4$ - are the dimensionless parameters.

3.1. Numerical procedure of solving the algebraic system

Then, the numerical method is applied to the system (7), which describes the problem of pipeline oscillations [17, 19-21]. Based on this method, an algorithm for the numerical solution of system (7) is described. By integrating system (7) two times over t, writing it in integral form and using a rational transformation, the singularities of the integral operators R^* and R_1^* are excluded. Then, setting $t=t_i$, $t_i=i\cdot\Delta t, i=1,2,...$ ($\Delta t = const$) and replacing the integrals with the quadrature trapezoidal formulas to calculate $w_{ik} = w_k(t_i)$, we get the formulas for the Koltunov-Rzhanitsin kernel $\left(R(t) = A\cdot\exp(-\beta t)\cdot t^{\alpha-1},\ \ 0<\alpha<1\right)$.

Thus, according to the numerical method for the unknowns, a system of algebraic equations is obtained [21-25]. To solve the system, the Gauss method is used. On the basis of the developed algorithm, a package of applied computer programs has been created. The results of calculations are presented in Table and reflected in graphs, Figures 3 and 4.

4 Numerical results and discussion

Results of calculations are presented in the table. The table shows the critical gas flow rates determined by formula (7). At rates, when $u>u_{cr}$, the oscillatory motion occurs with intensely increasing amplitudes and can cause the collapse of the structure, and in the case when $u<u_{cr}$, the oscillation amplitude attenuates. Note that for $u>u_{cr}$, the expansion of (7) diverges. Here, the u_{cr} is the critical rate of two-phase slug flow.

The study of the effect of viscosity is given. Calculations have shown that an account of viscous resistance leads to 40% decrease in the critical flow rate compared with the elastic solution. At A= 0 and A = 0.1, the critical rate of gas flow is 2.89 and 1.73, respectively. Studies have shown that in the special case the results of numerical modeling are consistent with the results obtained in [26]. With an increase in singular parameter α, the critical rate of the gas flow increases. This effect is more noticeable at α = 0.75, than at α =0.1. Numerical results show that the effect of the damping parameter β in the heredity kernel on the critical flow rate, as compared with the viscosity parameter A and the singularity parameter α, is insignificant. With an increase in this value, the flow rate decreases, but only slightly. The obtained value of the critical flow rate for a viscoelastic pipe at β = 0.07, is only 3.9% less than the values of the flow rate at β = 0.01.

An increase in parameter k_w leads to a significant change in the critical flow rate for the gas phase. Studies have been performed at k_w = 0; 10; 30 and 40.

Table 1. Dependence of the critical flow rate of a two-phase slug fluid on physico-mechanical and geometrical parameters of pipelines

A	α	β	γ_1	k_w	L_1	A_1	α_1	β_1	N_0	u_L	u_{gcr}
0 0.001 0.01 0.1	0.25	0.05	0.005	3.5	0.3	0	0.25	0.05	0.01	1.5	2.89 2.874 2.79 1.73
0.01	0.1 0.5 0.75	0.05	0.005	3.5	0.3	0	0.25	0.05	0.01	1.5	2.682 2.81 2.82
0.1	0.15 0.5 0.75	0.05	0.005	3.5	0.3	0	0.25	0.05	0.01	1.5	1.45 2.1 2.3
0.1	0.25	0.01 0.07	0.005	3.5	0.3	0	0.25	0.05	0.01	1.5	1.78 1.71
0.1	0.25	0.05	0.005	0 10 30 40	0.3	0	0.25	0.05	0.01	1.5	1.7 1.95 2.4 2.6
0.1	0.25	0.05	0.005	3.5	0.1 0.5 0.7	0	0.25	0.05	0.01	1.5	1.7 1.66 1.59
0.1	0.25	0.05	0.005	3.5	0.3	0.001 0.1 0.2	0.25	0.05	0.01	1.5	1.72 1.61 1.56
0.1	0.25	0.05	0.005	3.5	0.3	0.1	0.05 0.3 0.7	0.05	0.01	1.5	1.52 1.64 1.74
0.1	0.25	0.05	0.005	3.5	0.3	0.1	0.25	0.05	0.1 2.3 5.5 10.5	1.5	1.76 2.3 2.9 3.67

It is seen that with an increase in density of earth bases the critical gas flow rate increases.

The effect of external tensile forces in longitudinal direction of the pipeline has been studied. The table shows that an increase in the tensile forces in longitudinal direction of the pipeline leads to an increase in the critical flow rate for the gas phase. At N_o = 0.1 and N_o = 10.5, the critical flow rate for the gas phase is 1.76 and 3.67, respectively. On the contrary, compressive forces N_o lead to the same proportional reduction of the critical flow rate for the gas phase.

The table shows that an increase in the value of the viscosity parameter A_1 of bases leads to a decrease in the flow rate. Let's study the effect of the singularity parameter α_1 of the earth bases on the flow rate. With an increase in parameter α_1 from 0.05 to 0.7, the difference in critical rates determined by formula (7) increases by 14.5%. For example, at α_1 = 0.05 the flow rate is 1.52, and at α_1 = 0.7 the flow rate is 1.74.

The effect of the parameter $\overline{L}_1$ characterizing the length of the gas bubble zone on the critical flow rates for the gas phase is investigated. It is found that with an increase in the parameter $\overline{L}_1$, the critical flow rates for the gas phase decrease, which is explained by the fact that with an increase in the length of gas bubble zone the fluid rate in the gas bubble zone is much less than in the liquid slug zone, especially when the length of the pipe is large.

The effect of the viscoelastic properties of material on the pipeline behavior is investigated. Figure 3 shows the law of distribution of the pipeline deflection with account of viscoelastic properties of material and its development over time. For elastic pipelines the oscillations are almost periodic. As we see, an account of viscoelastic material properties of the structure sharply decreases the amplitude of oscillations. Meanwhile, the effect of the viscoelastic properties of pipeline material on the amplitude of its oscillations at the beginning of the process (part of the curve *w*(t) in the range of 0≤t≤0.2) is manifested to a much lesser extent. Beginning from τ≥0,2, the viscoelastic properties of material significantly affect the oscillatory process of the pipeline. Analysis of the results shows that an increase in the value of the viscosity parameter *A* leads to a damping of the oscillatory process. These conclusions and results are fully consistent with the conclusions and results in [1, 21, 26, 27].

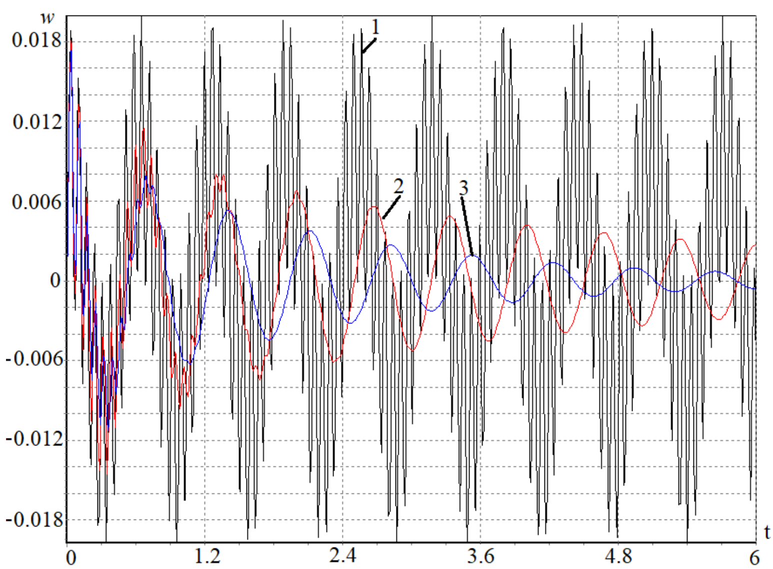

Fig. 3. Dependence of the pipe deflection *w* on time t at various parameters of viscosity: A=0 (curve 1); *A*=0.05 (curve 2); A=0.1 (curve 3); α=0.25; β=0.05; $k_w = 3.5$; $\overline{L}_1 = 0.3$; $\gamma_1 = 0.005$; $A_1 = 0.01$; $\alpha_1 = 0.25$; $\beta_1 = 0.05$; $N_o = 0.01$; $u_L = 0.3$; $u_g = 0.5$.

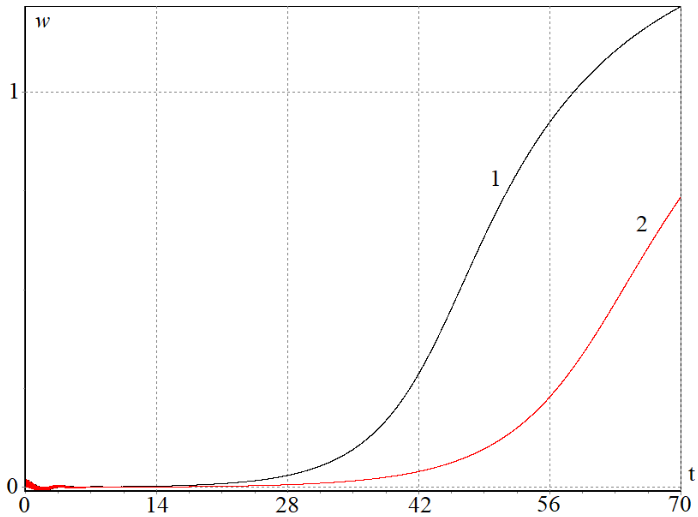

Fig. 4. Dependence of the pipe deflection w on time t at k_w =4.8 (curve 1); k_w =5.1 (curve 2); A=0.01; α=0.25; β=0.05; $\bar{L}_1 = 0.3$; $\gamma_1 = 0.05$; $A_1 = 0.01$; $\alpha_1 = 0.25$; $\beta_1 = 0.05$; $N_o = 0.01$; $u_L = 1.5$; $u_g = 2.81$.

Studies of the effect of the base parameter k_w on the oscillatory process (Figure 4) are given. As seen from the graph, dynamic instability is observed for both values of earth base k_w = 4.8 (curve 1) and k_w = 5.1 (curve 2) at the rate $u_L = 1.5$ and u_g =2.81, the motion is the oscillations with rapidly increasing amplitudes. The following parameters have been used in the calculation: A=0.01; α=0.25; β=0.05; $\bar{L}_1 = 0.3$; $\gamma_1 = 0.05$; $A_1 = 0.01$; $\alpha_1 = 0.25$; $\beta_1 = 0.05$; $N_o = 0.01$.

5 Conclusions

A mathematical model of the dynamics of a straight viscoelastic pipeline conveying two-phase slug flow has been developed. A computational algorithm has been developed for solving the problems of the dynamics of viscoelastic pipelines with conveying two-phase slug flow. On the basis of the developed computational algorithm, a package of applied computer programs has been created; it makes possible to investigate the oscillatory processes of viscoelastic pipelines conveying gas-containing two-phase slug flow. When modeling nonlinear problems, a number of dynamic effects have been investigated:

- it was established that the viscoelastic properties of the pipeline material lead to a decrease in the critical flow rate of the gas-containing two-phase fluid;
- it was found that an increase in the length of the gas bubble zone leads to a decrease in the amplitude and frequency of the pipeline oscillations;
- it was shown that an account of viscoelastic properties of earth bases leads to a decrease in the critical flow rate;
- it was found that an increase in the density of earth bases leads to an increase in the critical flow rate of the gas phase.

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