Spatial forced oscillations of axisymmetric inhomogeneous systems

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Abstract. The aim of this paper is to develop an adequate mathematical model, methods and algorithms for solving three-dimensional problems for axisymmetric spatial inhomogeneous viscoelastic systems (shells, foundations and bases) and to assess the dynamics of protective shell (containment) of a nuclear power plant (NPP) under resonant modes of vibration. The problem is solved using the semi-analytical finite element method. Firstly, the eigenmodes of vibration of the system are determined in an elastic three-dimensional statement, secondly, the solution to the problem of forced vibrations of viscoelastic systems is constructed using the expansion of these eigenmodes of vibration. Viscoelastic properties of the material are described using the hereditary Boltzmann-Volterra theory. The principle of virtual displacements is used to simulate dynamic processes in inhomogeneous viscoelastic systems. The convergence and accuracy of the solutions obtained are investigated by test problems. The frequency response characteristics (FRC) in various points of the NPP containment are estimated at various viscosity parameters of the material. It was revealed that the highest amplitude of vibrations in resonance modes occurs at close values of the frequency of external effect to the first eigen frequencies of the system; in the presence of dense spectra of eigen frequencies of the system, the highest amplitudes can occur at higher frequencies of external effect.

1 Introduction

High rates of construction, the erection of unique structures and buildings, the need to fulfill complex industrial orders require further development of the theory of calculating spatial axisymmetric structures under the influence of various loads, taking into account inelastic properties of the material.

The vibration intensity of real structures during earthquakes significantly depends on the degree of energy dissipation in them. It can be expected that the higher the energy dissipation in the structure, the less intense the resonant vibrations at a given level of excitation.

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There are various methods and means of dealing with unacceptable structure vibrations aimed to detune from resonances, one of which is to increase the damping properties of the material.

The assessment of dissipative properties of a structure as a whole is a rather difficult problem of the dynamics of deformable rigid body even for linear mechanical systems. For a complete assessment of dissipative properties of a structure, it is necessary to study its natural, steady-state and transient structural vibrations, taking into account internal friction in the material. The difficulty of solving this problem is, firstly, due to the lack of models more or less realistically describing the phenomenon of internal friction in the material. Secondly, solving the problem with well-known models leads to a number of tasks that are difficult to implement even on modern computers, due to the lack of computational methods and algorithms that meet a number of requirements for the tasks posed.

As is known, the first attempts at a theoretical description of dissipative properties of the materials are associated with the names of Vogt, Maxwell and Kelvin. However, at present, the most general model, reflecting all the features of material strain under various effects is the hereditary Boltzmann-Volterra theory of viscoelasticity; it describes, along with elastic properties, the dissipative properties (internal friction) of the material.

In engineering practice, various unique spatial axisymmetric structures of complex geometry are widely used. Such structures, in particular, include protective shells of nuclear power plants (NPP), cooling towers, smokestacks and ventilation pipes of nuclear and thermal power plants (NPP and TPP). To ensure reliable operation of such structures under various loads, first of all, it is necessary to evaluate their dynamic behavior under various effects, considering non-elastic behavior of their material. The solution of such problems using modern models that describe the dissipative properties of the material is an independent and rather difficult task.

Along with this, in the available publications, simple models of structures are often used as a calculation model for such structures; these simple models do not take into account such features as real geometry, design features of structures and dissipative properties of their material, which have a direct impact on the value of dynamic behavior of structures.

When assessing the stress-strain state and dynamic behavior of inhomogeneous axisymmetric systems, such as the protective shells of nuclear power plants, attention is often paid to elastic properties of the building material only.

For instance:

- in [1], a frequency equation was derived. The analysis of frequency and modes of vibrations of a shell was conducted. Using the asymptotic method, approximate frequency equations and simple calculation formulas were obtained that allow finding the values of the minimum natural frequencies of oscillations of the systems under consideration;
- calculation models of protective shells were developed in [2,3], taking into account the actual location of rod-like and prestressed reinforcement, as well as improved calculation algorithms, which made it possible to establish the main causes of the appearance and growth of tensile stresses in the radial reinforcement of the walls of protective shells;
- in [4], an equation was obtained for determining the resonant frequencies of axisymmetric vibrations of a hollow isotropic elastic ball. General solution of the vector equation of motion of the three-dimensional theory of elasticity in a spherical coordinate system was used;
- in [5], a technique was tested for determining, by technogenic vibrations, the dynamic characteristics of the reactor compartment of power unit No. 1 of Kalinin NPP. A comparison of results with well-known ones showed that the prevailing frequencies in vibration spectra correspond to the first mode of vibration, as a rigid body on an elastic base;

- thestudies in [6] give comparative results of experimental models of dynamic characteristics of protective shells, modern calculations of dynamic characteristics and the results of field studies;
- in [7,8], the eigenfrequencies of a cylindrical shell were investigated taking into account the thickness changes under various boundary conditions.

Along with this, the dynamic behavior and stress-strain state (SSS) of various structures, with account for features and operating conditions, were studied in [9-23].

These are just some of the publications in which the dynamics of various systems and structures are evaluated in different ways, and each theory or method used has its advantages and disadvantages.

Therefore, the development of an adequate model, an effective technique and algorithm for assessing the dynamic behavior of inhomogeneous axisymmetric structures, taking into account their design features and dissipative properties of their material, is an urgent task in the mechanics of a deformable rigid body.

The goal of this work is to develop an adequate mathematical model, methods and algorithm for solving the problem of forced steady-state vibrations of viscoelastic axisymmetric structures using three-dimensional theory and to estimate the amplitude-frequency characteristics of protective shells of nuclear power plants under resonant modes of vibration.

2 Materials and methods

An inhomogeneous spatial ax symmetric system is considered (figure 1) consisting of anarch-like structure (protective shell) -1, foundation 2 and soil base 3, which occupy volumesV_1,V_2 and V_3,respectively. On the lower part of the system Σ_u, a periodic kinematic effect $\vec{u}_0(\vec{x},t)$ is set. It is necessary to determine the displacement components of the structure points of the system in question (figure1) in the resonant modes of oscillations at various frequencies of kinematic effect.

As is known, theforced steady-state oscillations of a system occur in the presence of external periodic impact. In this case, the initial conditions are not taken into account. The study of this type of system oscillations allows identifying the dependences of the maximum amplitudes of displacements and stresses at any point of the system under consideration (figure 1) on the system parameters and external influences. In this case, the dissipative properties of the system are manifested mainly in resonance modes. The values of resonance amplitudes of displacements and stresses are used as a quantitative estimate of the intensity of dissipative processes.

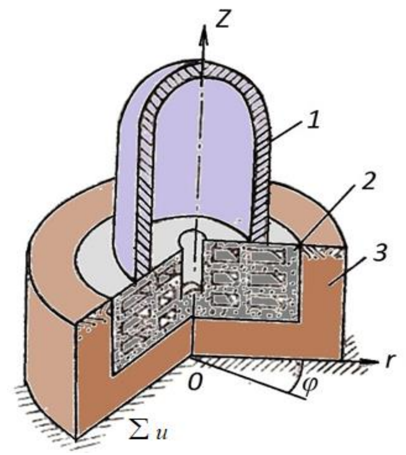

Figure 1. Inhomogeneous spatial axisymmetric system.

To simulate the strain process of the system under consideration (figure 1) under forced steady-state vibrations, the principle of virtual displacements is used, according to which the sum of all active forces acting on the system, including the inertia forces on virtual displacements, is zero, i.e.:

$$\delta A = -\int_{V_1} \sigma_{ij}\delta\varepsilon_{ij}\mathrm{d}V - \int_{V_2} \sigma_{ij}\delta\varepsilon_{ij}\mathrm{d}V - \int_{V_3} \sigma_{ij}\delta\varepsilon_{ij}\mathrm{d}V -$$

$$- \int_{V_1} \rho_1 \ddot{\vec{u}}\delta\vec{u}\mathrm{d}V - \int_{V_2} \rho_2 \ddot{\vec{u}}\delta\vec{u}\mathrm{d}V - \int_{V_3} \rho_3 \ddot{\vec{u}}\delta\vec{u}\mathrm{d}V = 0, \quad (1)$$

$$i, j = r, z, \varphi$$

Along with (1), to model the strain process, it is necessary to know:

- kinematic boundary conditions

$$\vec{x} \in \Sigma_u : \vec{u}_0(\vec{x}, t) = \vec{\psi}_1(t), \quad (2)$$

- Cauchy relation connecting the strain tensors with the components of displacement vector [24]

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \ \varepsilon_{\varphi\varphi} = \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u}{r}, \ \varepsilon_{zz} = \frac{\partial u_z}{\partial z},$$

$$\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right), \quad (3)$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right), \ \varepsilon_{\varphi z} = \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right),$$

- the ratio of the generalized Hooke's law, connecting stress tensors σ_{ij} with strain tensors ε_{ij} of the form [24]:

$$\sigma_{ij} = \widetilde{\lambda}_n \varepsilon_{kk}\delta_{ij} + 2\widetilde{\mu}_n \varepsilon_{ij} \quad (4)$$

$$i, j, k = r, z, \varphi$$

In the case when the material of the *n*-th element of the system is elastic, quantities $\tilde{\lambda}_n$ and $\tilde{\mu}_n$ are the Lamé constants; if the material has viscoelastic properties, then $\tilde{\lambda}_n$ and $\tilde{\mu}_n$ are the Volterra integral operators and have the following form [16,24, 25]:

$$\left.\begin{aligned} \tilde{\lambda}_n\varphi &= \lambda_n\left[\varphi(t) - \int_{-\infty}^{t} \Gamma_{\lambda_n}(t-\tau)\varphi(t)d\tau\right] \\ \tilde{\mu}_n\varphi &= \mu_n\left[\varphi(t) - \int_{-\infty}^{t} \Gamma_{\mu_n}(t-\tau)\varphi(t)d\tau\right] \end{aligned}\right\} \tag{5}$$

Here $\Gamma_{\lambda_n}, \Gamma_{\mu_n}$ - are the kernels of relaxation; $\varphi(t)$- an arbitrary function of time; $\vec{u}, \sigma_i, \varepsilon_{ij}$ - the components of the displacement vector, stress and strain tensors, respectively; ρ_n - the density of the material of the system; λ_n and μ_n - the Lame constants; $\delta\vec{u}, \delta\varepsilon_{ij}$ -isochronous variations of displacements and strains; $\vec{\psi}_1(t)$- periodic function of time; the index n = 1,2,3 means the individual part of the system (i.e. construction, foundation, base) to which this characteristic relates; δ_{ij}– the Kronecker symbol; $\vec{x} = \{r, z, \varphi\}$ - cylindrical coordinates; $\vec{u} = \{u_r, u_z, u_\varphi\}$ - isplacement vector components.

Now the problem of forced steady-state vibrations of the system under consideration (figure 1) is reduced to determining the fields of displacements $\vec{u}(\vec{x},t)$ and stresses $\sigma_{ij}(\vec{x},t)$ in the system arising under kinematic effects (2), satisfying equations (1), (3), (4), (5) and conditions of periodicity at any virtual displacement $\delta\vec{u}$.

Next, the solution to variational problem (1), (3), (4), (5) is sought in the form of an expansion in modes of natural vibrations [16] of elastic system (figure 1):

$$\begin{aligned} \vec{u}(\vec{x},t) &= \vec{u}_0(\vec{x},t) + \sum_{k=1}^{N} \vec{u}_k^*(\vec{x}) y_k(t), \\ \delta\vec{u} &= \sum_{k=1}^{N} \vec{u}_k^*(\vec{x}) \delta y_k(t), \end{aligned} \tag{6}$$

Where $\vec{x} = \{r,\ z,\ \varphi\}$ –are the cylindrical coordinates; $\vec{u} = \{u_r, u_z, u_\varphi\}$components of displacement vector; $\vec{u}_0(\vec{x},t)$-known function (2) satisfying the boundary conditions of the problem; $\vec{u}_k^*(\vec{x}) = \{u_{kr}^*(\vec{x})\ , u_{kz}^*(\vec{x})\ , u_{k\varphi}^*(\vec{x})\}$- eigenmodes of vibrations of elastic system (figure 1); $y_k(t)$ - the sought for functions of time; $\delta y_k(t)$ - arbitrary constants; *N*- the number of eigenmodesheld in expansion (6).

The eigenmode of vibrations -$\vec{u}_k^*(\vec{x})$ of elastic system (figure 1) is found by the finite element method (FEM). To find the eigenmodes of the system's vibrations (figure 1), the semi-analytical finite element method is used, i.e. in one coordinate (in φ), a solution is sought using trigonometric functions; in two other coordinates (r, z), the solution is sought by the finite element method using an elemental annulus of a triangular section.

In the case of forced steady-state oscillations, the integral operators (5) are replaced by relations of the form

$$\left.\begin{aligned}\widetilde{\lambda}_n\varphi = \lambda_n\left[1-\Gamma_{\lambda n}^{C}(\Omega)-i\Gamma_{\lambda n}^{S}(\Omega)\right]\varphi\\ \widetilde{\mu}_n\varphi = \mu_n\left[1-\Gamma_{\mu n}^{C}(\Omega)-i\Gamma_{\mu n}^{S}(\Omega)\right]\varphi\end{aligned}\right\}, \tag{7}$$

here:

$$\Gamma_{\lambda_n}^{c}(\omega_R)=\int_0^\infty \Gamma_{\lambda_n}(\tau)\cos\omega_R\tau d\tau,\ \Gamma_{\mu_n}^{c}(\omega_R)=\int_0^\infty \Gamma_{\mu_n}(\tau)\cos\omega_R\tau d\tau,$$

$$\Gamma_{\lambda_n}^{s}(\omega_R)=\int_0^\infty \Gamma_{\lambda_n}(\tau)\sin\omega_R\tau d\tau,\ \Gamma_{\mu_n}^{s}(\omega_R)=\int_0^\infty \Gamma_{\mu_n}(\tau)\sin\omega_R\tau d\tau,$$

$\Gamma_{\lambda_n}^{S}, \Gamma_{\lambda_n}^{C}\ \Gamma_{\mu_n}^{S}, \Gamma_{\mu_n}^{C}$ - are the sines and cosines of the Fourier image of the kernel $\Gamma_{\lambda n}(\tau), \Gamma_{\mu n}(\tau)$s.

Next, consider the case when the kinematic effect acts on the base Σ_u of the system (figure 1)

$$\vec{x}\in\Sigma_u : u_{0r} = a_0\cos\varphi^*\sin pt,\ u_{0\varphi} = a_0\sin\varphi^*\sin pt,\ u_{0z} = 0 \tag{8}$$

Herea_0 - is the amplitude, p is the frequency of external effect.

Substituting (6), (1) into (3), (4) and (5), performing integration over the volume V and equalizing to zero the factors at independent variations of δy_k, lead to a system of linear integro-differential equations with respect to the sought for functions $y_k(t)$. If toconsider that the volume strain of the system occurs according to elastic law(K_n), and shear strain (μ_n) according to viscoelastic law, then we obtain the following system of equations

$$\sum_{l=1}^{N}\left[A_{kl}\ddot{y}_l(t)+B_{kl}y_l(t)-C_{kl}\int_{-\infty}^{t}\Gamma_{\mu n}(t-\tau)y_l(\tau)dr\right]=a_k\sin pt, \tag{9}$$

$k = 1,2,\dots N$

where coefficientsA_{kl}, B_{kl}, C_{kl}, $a_k$are determined in the process of integration over volume V of functions $\vec{u}_k^*(\vec{x})$and their scalar products, entering equation (1); K_n, μ_n - are the instantaneous volume and shear moduli of elasticity.

The exact solution to the system of integro-differential equations (9) is sought in the form

$$y_l = a_l\sin pt + b_l\cos pt \tag{10}$$

wherea_l, b_l- are the sought for constants. Substitution of (10) into system (9) leads to a system oflinear algebraic equations for the sought for constants a_l, b_l of the form

$$\sum_{l=1}^{N}\left[a_l\left(-P^2A_{kl}+B_{kl}+C_{kl}\Gamma_{\mu_n}^{c}\right)+C_{kl}\Gamma_{\mu_n}^{s}b_l\right]=a_k,$$

$$\sum_{l=1}^{N}\left[-C_{kl}\Gamma_{\mu_n}^{s}a_l+b_l\left(-P^2A_{kl}+B_{kl}+C_{kl}\Gamma_{\mu_n}^{c}\right)\right]=0 \tag{11}$$

from the solution of which a_l, $b_l$are determined at$l$=1,2,…, N.

Here $\Gamma^{s}_{\mu_n}$, $\Gamma^{c}_{\mu_n}$ are the sine and cosine of the Fourier images of relaxation kernel $\Gamma^{s}_{\mu_n}(\tau)$.

Holding various numbers of Neigenmodes of vibration in (6), we can determine the components of the displacement vector $\bar{u}=\{u_r,u_z,u_\varphi\}$ at an arbitrary point in the system (figure 1). For this, at various frequencies "p" of external action it is necessary to solve equation (11), and the results of solution must be included in (10) and (6).

3 Results and Discussion

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As a result of solving this problem, the amplitude-frequency characteristics (AFC) were constructed for various points of structure under different frequencies "p". For different points of structure and for various components of the displacement vector $\bar{u}=\{u_r,u_z,u_\varphi\}$, the AFC were constructed at ~ 150 values of frequency "p" of external influences (8); these values of frequency "p" were located more densely in the assumed vicinity of viscoelastic resonances. Then, the obtained (constructed) amplitude-frequency characteristics for various points of the structure were analyzed to determine the resonant amplitudes and frequencies of viscoelastic system in the resonant mode of oscillation.

As is known, AFC is an indicator of how certain points of the structure react to certain effects at certain viscosity parameters under resonant modes of vibration.

The accuracy of the results obtained was studied while holding in expansion (6) various eigenmodes of elastic system.

When performing specific calculations, elastic module$E$ of the foundation and the base weretaken two orders of magnitude greater than the elastic modulus of protective shells of the system (figure 1).

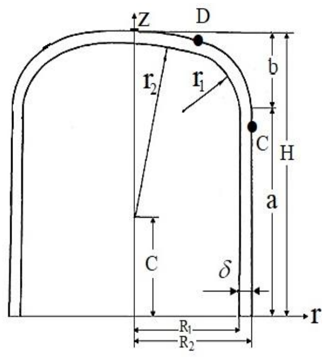

Figure 2. Geometrical parameters of a structure (protective shell).

The material of protective shell (figure 1) is hypothetical, i.e.: K=1.45; μ=*0.78*,ρ=1/150.

The relaxation kernel is taken in the form [16, 23, 24]: $\Gamma(t-\tau)=\dfrac{Ae^{-\beta(t-\tau)}}{(t-\tau)^{1-\alpha}}$, with parameters:

1) α=0; A=0.008; β=0.003 (low viscosity);
2) α=0.1; A=0.040; β=0.003 (high viscosity).

Geometrical dimensions of the shell (figure 2) in dimensionless form are taken as:H/R_2=2.346;R_1/R_2=0.949; r_1/R_2=0.54; r_2/R_2=1.468; a/R_2=1.662; b/R_2=0.684; c/R_2=0.827; δ/R_2=0.0506.

Here: K, μ –are the instantaneous volume and shear moduli of elasticity.

Figure 3 shows the convergence of calculation results of AFC for point *D* for displacement component u_r while holding different number *N* ofeigenmodes of vibration of elastic system in expansion (6). As the results show, to obtain the AFC with acceptable accuracy, it is necessary to hold at least 5 - 6 terms in expansions (6). If tohold less number of terms N, the result is far from the true one, i.e. large discrepancies are observed in the obtained solutions. This is clearly seen in figure 3.

The reliability of the results obtained is also proven by the onset of elastic resonance at the coincidence of impact frequency “*p*” with any of the frequencies of natural vibrations, i.e.: ω_1 = 0.03437; ω_2 =0.10082; ω_3 =0.12174; ω_4 =0.15619; ω_5 =0.19758 (the material is considered hypothetically, therefore, the frequency dimension is not set). The characteristic response of the system when the frequencies of external impact “*p*” coincide with the frequency of natural vibrations confirms the reliability of the developed model, methods and algorithm.

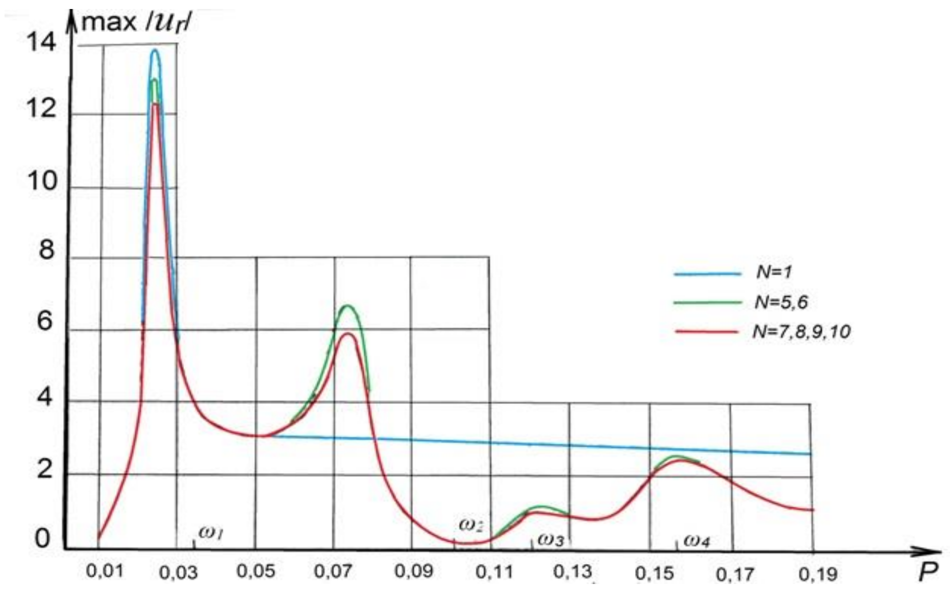

Figure 3. AFC of the structure at /u_r/ in point D (figure 2) at high viscosity of the material.

The convergence of the solution was studied at high and low viscosities, with an increase in the number of eigenmodes *N* in expansion (6) of the sought for solution. It turned out that in the case of low viscosity, the addition of two new eigenmodes to the solution leads to the appearance of new peaks in the resonance curve without significant distortion of the curve far from the eigenfrequencies of the newly added eigenmodes. The results obtained at low and high viscosities of the material coincide qualitatively, differing only in greater amplitudes in the vicinity of resonances at low viscosity.

Figure 4 shows, as an example, the AFC for the component of displacements u_r obtained at high viscosity of the material at one point (point C (figure 2)) of the structure,

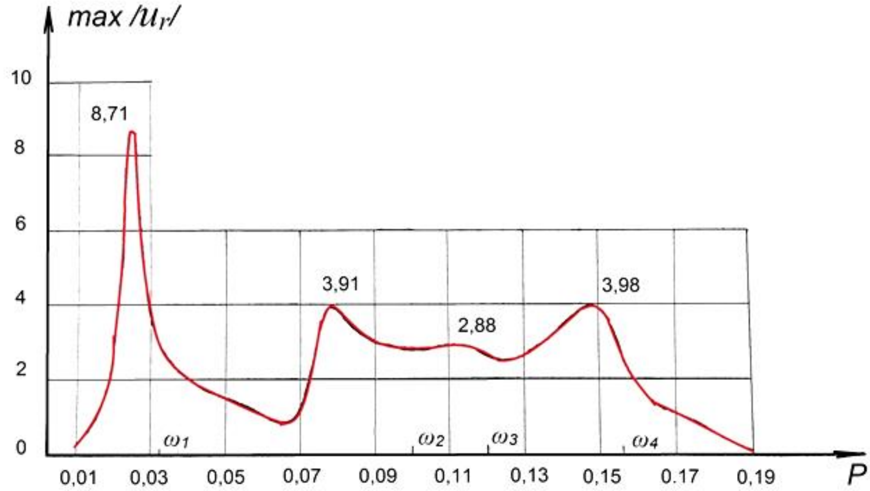

Figure 4. AFC of the structure at/u_r/ at point C (figure 2) at high viscosity of the material.

As seen from these figures, structural vibrations occur in the first, second, and third modes of vibrations. The mutual influence of various modes of vibrations on structure behavior, even at high viscosity, is not observed. This is apparently due to the non-dense spectrum of natural frequencies of the structure.

4 Conclusions

1. A mathematical model, methods and algorithm for the study of forced steady-state oscillations of inhomogeneous spatial axisymmetric systems in three-dimensional settings were developed.
2. The reliability of the developed methods and algorithms was validated by test problems, that is, by comparing,in elastic case,the resonant frequencies of external influences with the eigenfrequencies of the system.
3. The convergence of the obtained results of AFCwas validateddepending on the number of held eigenmodes of elastic system.
4. The steady-state forced oscillations of spatial inhomogeneous viscoelastic axisymmetric systems were studied in a three-dimensional statement in resonant modes of vibration.
5. The constructed frequency response characteristics for different points of inhomogeneous viscoelastic axisymmetric systems show that the highest amplitude of oscillations in the resonant mode occurs at close values of external effect frequency to the first eigenfrequencies of a structure; and in the presence of dense eigenfrequency spectra, the highest amplitudes can occur at higher frequencies of external effect.

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