Application of quasilinear and CGA models for designing significantly nonlinear control systems

R Neydorf[1], *A* Gaiduk[2,*], *N* Kudinov[1], *V* Dolgov[1]

[1]Don State Technical University, 1, Gagarina sq., Rostov-on-Don, 344003, Russia
[2]Southeren Federal University, Taganrog, Russia

Abstract. To create control systems, mathematical models of objects are required, which often have to be obtained experimentally. In this case, the numerical data of experiments on the identification of essentially nonlinear objects can be satisfactorily approximated only in certain areas, which leads to fragmentary models that are not analytical. In these cases, when only fragmentary models are adequate, it is proposed to apply the new Cut-Glue approximation method, which allows obtaining a model with analytical properties. The proposed approach to the unification of the Cut-Glue approximation method is demonstrated by solving the problem of synthesizing a nonlinear airship altitude control system and studying it.

1 Introduction

The development of modern automatic control systems (ACS) for various technological processes, in particular, transport, chemical, information, telecommunications objects and their systems is impossible without their mathematical models (MM). Analytical methods for the synthesis of ACS are a developed tool and make it possible to find the laws of control of objects, including nonlinear ones, by solving systems of resolving equations, which take into account both the requirements for the quality of the ACS and the properties of control objects [1-4]. It is for the compilation of resolving equations, taking into account the requirements for the quality of control, that mathematical models of control objects are needed in one of the special forms, in particular, matrix-vector.

As applied to nonlinear control objects with the connection topology of elements unknown to the researcher, an experimental approach is usually used to construct their mathematical models. It is based on comparing the test actions fed to the inputs of the object with the reactions to these actions [2, 5-7]. The obtained experimental data are approximated by suitable mathematical functions. In the case of complex systems and processes, experimental data are distributed nonlinearly, so it is difficult to derive the approximating function, and it is impossible to derive without computing. In these cases, a "piecewise" (fragmentary) approach is used [6-8]. Such fragmentary mathematical models describe experimental data rather accurately, but they are not analytical. New Cut-Glue

* Corresponding author: gaiduk_2003@mail.ru

Approximation Method (CGA) allows you to additively and then multiplicatively combine fragmentary models into a single analytical CGA model [7, 8]. The study of the structure and properties of CGA-models showed that they provide the possibility of analytical synthesis of ACS. At present, many analytical methods for the synthesis of nonlinear ACS have been developed, such as the method of linearization by feedback [9], the method of point transformations [1, 4, 10], the method of inverse step [11], the method of passivation [12], output control [13], method of quasilinear models [10]. However, application of these methods is possible after conversion of mathematical model of control object into one of many special forms of equation representation. Precise transition to such forms without loss of adequacy is possible for analytical MM, i.e., the system of differential equations, the right part of which can be represented by a convergent Taylor series. For example, the method of linearization by feedback involves the use of Lee algebra, which requires finding partial derivatives from the right parts of the differential equations of MM, similar requirements for MM occur in other methods. The study of this problem showed that the process of synthesis of nonlinear control systems by analytical methods is well formalized, as well as for linear ones, but the initial MM must be analytical. Let us consider the main provisions of the ACS synthesis method based on quasilinear models of the control object, as well as the CGA method.

2 Problem formulation

The complex problem of synthesizing nonlinear control laws for an essentially nonlinear control object is supposed to be solved by the example of synthesizing an airship altitude control system in the following sequence: first of all, the features of the method for solving the problem of synthesizing nonlinear ACS based on quasilinear MMs are considered in order to clarify the requirements for MM caused by this method; then – the process of building a CGA model of the airship altitude control process and its transformation into a quasilinear form; synthesis of the law for controlling the height of the airship flight; and, finally, numerical simulation of the synthesized ACS to confirm the results of the proposed approach, especially in the areas of fragments joining.

3 Synthesis of a control system by method of quasilinear models

Quasilinear MM of nonlinear control objects is a system of differential equations of the following form:

$$\dot{x} = O(x)x + d(x)u \tag{1}$$

where $\boldsymbol{x}$ is n-vector of state variables; $O(x)$ and $d(x)$ is functional matrix and vector whose elements $o_{ij}(x)$ and $h_i(x)$ depend on state variables; $\boldsymbol{u}$ is scalar control.

The required control law for an object described by a quasilinear model (QLM) (1) is searched for in the following form

$$u \equiv u(x) = -k^T(x)x = -[k_1(x) \quad k_2(x) \quad \dots \quad k_n(x)]x \tag{2}$$

Substitution of (2) into (1) shows that the MM of a closed ACS is also a QLM:

$$\dot{x} = S(x)x \tag{3}$$

where $S(x)$ is also a functional system matrix of the same dimension as $O(x)$, the elements $d_{ij}(x)$ of which are functions of state variables. Moreover, the matrix $S(x)$ of the closed-loop system is determined by the expression

$$S(x) = O(x) - d(x) \cdot k^T(x) \tag{4}$$

We emphasize that the system matrix of a closed ACS (3), like any other square matrix, has a characteristic polynomial:

$$S(\lambda, x) = \det(\lambda E - S(x)) \tag{5}$$

which, based on the properties of determinants, can be represented as the sum of two polynomials:

$$S(\lambda, x) = O(\lambda, x) + \sum_{i=1}^{n} k_i(x) \cdot V_i(\lambda, x) \tag{6}$$

where

$$O(\lambda, x) = \det(\lambda E - O(x)) = \lambda^n + o_{n-1}(x)\lambda^{n-1} + \ldots + o_1(x)\lambda + o_0(x) \tag{7}$$

$$V_i(x, \lambda) = e_i \, \mathrm{adj}(\lambda E - O(x)) \, d(x) = \sum_{j=0}^{n-1} v_{ij}(x) \lambda^j \tag{8}$$

In expressions (6) – (8) λ is complex variable; $o_i(x)$ is functional coefficients of the characteristic polynomial $O(x)$ of the control object matrix (1); e_i is *i*-th row of the identity *n*×*n*-matrix E; $\mathrm{adj}(\lambda E - O(x))$ is the adjoint matrix for the matrix ($\lambda E - O(x)$) [14]; $v_{ij}(x)$ are the functional coefficients of the polynomials $V_i(x), i = \overline{1, n}$.

When synthesizing nonlinear closed-loop control systems of type (3), first of all, the problem arises of ensuring the stability of their equilibrium position $x = 0$. It is most convenient to solve this problem by the method of Lyapunov functions, but the construction of Lyapunov functions for nonlinear systems of general form is still a difficult problem. But since the form of QLM of nonlinear systems is similar in structure to linear systems, it is logical and the desired characteristic polynomial of a closed system to give the form of a polynomial with constant coefficients, which is stable, i.e. satisfies, for example, the Hurwitz criterion. This makes it possible to study the stability of the equilibrium position of the CLM of a closed system using the Lyapunov function in the form of a quadratic form, which is used to study the stability of linear perturbed systems [10].

To implement this approach, it was proposed in equality (7) to replace the polynomial $S(\lambda, x)$ by a polynomial $S^*(\lambda)$ of the n^{th} degree with constant coefficients, which leads equality (7) to an equivalent polynomial equation for unknown coefficients $k_i(x),\ i = \overline{1, n}$. On the other hand, it was shown in [15] that solving a polynomial equation reduces to solving a system of algebraic equations. As a result, it is possible not only to find the required control law (2) providing stability in the vicinity of the equilibrium position, but also to establish the conditions of solvability of the nonlinear systems synthesis problem by this method. The stability of the equilibrium position of the synthesized quasilinear systems was proved by the method of Lyapunov functions in [10]. It was found in the course of the proof that by changing the polynomial $D^*(\lambda)$ coefficients the character of the transition process of the synthesized system can be varied, in particular, its duration.

In order to construct the above system of algebraic equations, the polynomial $S(\lambda, x)$ in equality (6) is replaced by the polynomial $S^*(\lambda)$, then the difference between the polynomials $S^*(\lambda)$ and $O(\lambda, x)$ and the coefficients of the polynomial of this difference are determined:

$$R(\lambda,x) = S^*(\lambda) - O(\lambda,x) = r_{n-1}(x)\lambda^{n-1} + r_{n-2}(x)\lambda^{n-1} + \ldots + r_1(x)\lambda + r_0(x) \quad (9)$$

As a result, polynomial equation (6) takes the form

$$\sum_{i=1}^{n} k_i(x) \cdot V_i(\lambda,x) = R(\lambda,x) . \quad (10)$$

By equating in (10) the coefficients at the same degrees λ on the left and right, as well as recording the obtained equality in vector matrix form, we obtain a system of algebraic equations:

$$\begin{bmatrix} v_{1,0}(x) & v_{2,0}(x) & \cdots & v_{n,0}(x) \\ v_{1,1}(x) & v_{2,1}(x) & \cdots & v_{n,1}(x) \\ \vdots & \vdots & \vdots & \vdots \\ v_{1,n-1}(x) & v_{2,n-1}(x) & \cdots & v_{n,n-1}(x) \end{bmatrix} \cdot \begin{bmatrix} k_1(x) \\ k_2(x) \\ \vdots \\ k_n(x) \end{bmatrix} = \begin{bmatrix} r_0(x) \\ r_1(x) \\ \vdots \\ r_{n-1}(x) \end{bmatrix} \quad (11)$$

The System (11) has a unique solution only if the QLM (1) of the object is completely controllable [10], that is, when the following controllability condition is met:

$$\det U(x) = \det[d(x)\ O(x)d(x)\ \ldots\ O^{n-1}(x)d(x)] \neq 0 \quad (12)$$

Expressions (7) – (9) and (11) are the main calculated relations of the method for the synthesis of nonlinear ACS using quasilinear MM. They allow for a nonlinear plant specified by a quasilinear model (1), when natural condition (12) is satisfied, to find the sought quasilinear control law (2).

To apply the above method, it is necessary to represent in the form of a quasilinear model of type (1) a given MM of a nonlinear control object. In this case, it is also necessary that all the state variables of this object be available for measurement using sensors or by some indirect methods. For nonlinear control objects, the quasilinear form of their MM can be obtained with the only restriction on the right-hand sides: they must be analytical functions. It is this property that makes it possible to impart to mathematical models of nonlinear objects or technical systems the use of the Cut-Glue approximation method over the entire range of variation of the experimental data arguments.

4 Fragmentary object modelling

Let us consider the features of the approach proposed in the work using the example of an airship flight altitude control system. The main feature of the airship flight altitude control process is the dependence of its lift on the angle of attack [16]. Using this effect allows you to significantly save energy resources of the airship's propulsion systems. This dependence is non-linear even at a constant longitudinal flight speed. In this regard, its linearization or a single regression description over the entire range of changes in the angles of attack is unpromising, since it is a priori easy to predict a large approximation error in the vicinity of the characteristic breaks. The problems associated with the adequacy of mathematical processing of identification data that precedes the synthesis of the control system do not end with the problems of approximating the aerodynamic characteristics, they extend to the designed and operational technical systems and their components.

Let us analyze the modeled system and its mathematical model. We will consider the walls of the airship to be solid and simulate its dynamics, allowing two degrees of freedom. This allows writing MM in the form of a system of four differential equations with respect to vertical displacement, its speed, pitch angle and rate of its change. All MM components, in addition to the dependence of the velocity on the force and, in turn, on the pitch angle, are linear and known, since they express the fundamental laws of conservation of momentum. The main problem is to take into account in the MM the nonlinear empirical dependence of the lift on the airship angle of attack, which in practice is determined on the basis of experimental data on blowing airship body models in a wind tunnel.

In general, the purpose of fragmentation of experimental data is to select a set of segments of the argument of nonlinear dependence and positions of their joining so that on each segment the approximating functions do not deviate much from the experimental data [6,7]. This criterion allows you to determine the points of a sharp change in the trend. The substantiation of the possibility of approximating a smooth function by a polynomial with constant coefficients on individual intervals, according to the authors, is given in the Weierstrass approximation theorem [8] on the possibility of approximating a continuous functional dependence of a polynomial. Let us take a closer look at the method of approximating experimental data when constructing the dependence of the lift force of an airship $F(\alpha)$ on the angle of attack α. Analysis of the experimental data leads to the conclusion that it is advisable to isolate three fragments when the angle α changes from –36° to 72°. The corresponding experimental data are shown in table 1. And in the corresponding figure 1, these values are indicated by markers "+".

From these data, it is clearly seen that in the investigated range of angles of attack, the static characteristic of the function $F(\alpha)$ is radically different in individual sections, joining at the "critical points" - α_{cr}^{1} and α_{cr}^{2}. Neighboring fragments in the vicinity of point $\alpha_{cr}^{1} = 0^{o}$ differ in the numerical value of the finite difference $\Delta F/\Delta\alpha$ to the right and left of it, and in the vicinity of point $\alpha_{cr}^{2} = 40,767^{o}$ these differences on fragments 2 and 3 have opposite signs. The decrease in lift with an increase in the angle at $\alpha > 40,767^{o}$ is most likely associated with a shift in the point of separation of the boundary layer when flowing around the airship body and its outer elements. Thus, the experimental static characteristic of an aircraft presented in table 1 could be well approximated by a function of a discontinuous derivative. This characteristic will be called non-linear and non-analytical.

Table 1. Segments of $F(\alpha)$.

1st fragment		2nd fragment		3rd fragment	
α^{o}	$F_1^e(\alpha)$	α^{o}	$F_2^e(\alpha)$	α^{o}	$F_3^e(\alpha)$
–36	–166.7	0.0	00.00	40.767	185.4
–30	–156.7	6	38.95	48	113.4
–24	–143.9	12	76.34	54	67.55
–18	–125.0	18	111.1	60	34.31

–12	–96.81	24	141.3	66	13.73
–6	–55.97	30	165.0	72	5.824
0	00.00	36	180.4		
		40.767	185.4		

As a result of approximation by polynomials, functions $F_1^e(\alpha)$, $F_2^e(\alpha)$ and $F_3^e(\alpha)$ are obtained, which describe the dependence of the original function $F(\alpha)$ on each interval of the argument separately. However, over the entire range of variation of the argument α, such a model is not an analytical function, since at the critical values of the angle $\alpha_{cr}^1 = 0^o$ and $\alpha_{cr}^2 = 40,767^o$, there are abrupt changes in the rate of change in $F(\alpha)$ with increasing angle α.

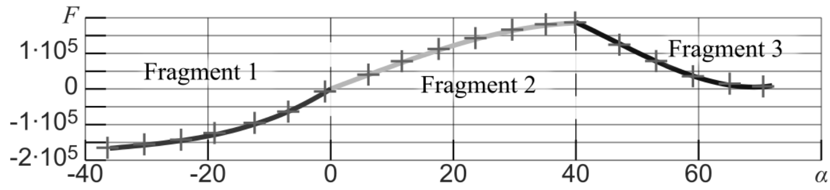

Fig. 1. Cut-Glue approximation of the airship characteristic.

A distinctive feature of the Cut-Glue approximation method, as shown in [7, 8], is its ability to impart analytical properties to a fragmentary dependence, provided that the functions approximating the fragments are analytic. In this case, the discontinuities of the derivatives at the critical points are described by impulses with an “adjustable” degree of smoothing.

In accordance with the CGA method, an analytical model of the "angle of attack - lift" channel is constructed at $-36^o \le \alpha \le 72^o$. In this case, the fragmentary model of the function $F(\alpha)$ in the specified range includes three fragments; therefore, the CGA-model constructed by the Cut-Glue approximation method [7] has the following form:

$$\begin{aligned} F(\alpha) = \big[& F_1(\alpha)\cdot E_1(\alpha,\alpha_{l1},\alpha_{r1},\varepsilon) + \\ & + F_2(\alpha)\cdot E_2(\alpha,\alpha_{l2},\alpha_{r2},\varepsilon) + \\ & + F_3(\alpha)\cdot E_3(\alpha,\alpha_{l3},\alpha_{r3},\varepsilon)\big]\cdot\alpha = F^L(\alpha)\cdot\alpha. \end{aligned} \tag{13}$$

where

$$\begin{aligned} F_1(\alpha) &= 10824,2 + 259,371\alpha + 2,42438\alpha^2, \\ F_2(\alpha) &= 6511,65 + 0,97905\alpha - 1,01816\alpha^2 - 0,00461\alpha^3, \\ F_3(\alpha) &= 43411 - 1701,57\alpha + 22,3751\alpha^2 - 0,0986265\alpha^3 \end{aligned} \tag{14}$$

In expression (13), the multiplicative isolating functions (MIF) $E_i(\alpha,\alpha_{li},\alpha_{ri},\varepsilon)$ introduced in accordance with the paradigm of the CGA method are defined as follows:

$$E_i(\alpha,\alpha_{li},\alpha_{ri},\varepsilon) = \frac{\left[\alpha - \alpha_{li} + R_l(\alpha,\alpha_{li},\varepsilon)\right]\cdot\left[\alpha_{ri} - \alpha + R_r(\alpha,\alpha_{ri},\varepsilon)\right]}{4\cdot R_l(\alpha,\alpha_{li},\varepsilon)\cdot R_r(\alpha,\alpha_{ri},\varepsilon)} \tag{15}$$

where $R_l\left(\alpha,\alpha_{li},\varepsilon\right)=\sqrt{(\alpha-\alpha_{li})^2+\varepsilon^2}$, $R_r\left(\alpha,\alpha_{ri},\varepsilon\right)=\sqrt{(\alpha_{ri}-\alpha)^2+\varepsilon^2}$, $i=1,2,3$.

In the case of the CGA model (13), the parameters of the MIF (15) have the following values: $\alpha_{l1}=-37^{\circ}$; $\alpha_{r1}=0^{\circ}$, $\alpha_{l2}=\alpha_{r1}$; $\alpha_{r2}=40,767^{\circ}$; $\alpha_{l3}=\alpha_{r2}$; $\alpha_{l3}=73^{\circ}$; $\varepsilon=0,01$. The α_{l1} and α_{r3} values are shifted by 1 degree to the left and right, respectively, to eliminate edge distortions of the simulated values of $F(\alpha)$. Function $E_i(\alpha,\alpha_{li},\alpha_{ri},\varepsilon)$ within the interval $\alpha_{li}<\alpha<\alpha_{ri}$ is practically equal to 1, at the boundaries, its values are $E_i(\alpha_{li},\alpha_{li},\alpha_{ri},\varepsilon)=E_i(\alpha_{ri},\alpha_{li},\alpha_{ri},\varepsilon)=0,5$, and beyond its boundaries is equal to zero [7, 8].

The graph of the resulting analytical function $F(\alpha)$, plotted in figure 1 according to expression (13), is a continuous curve, practically without error, coinciding with the sections of the interval characteristics $F_1^e(\alpha)$, $F_2^e(\alpha)$ and $F_3^e(\alpha)$. The fact that function (13) is continuous can be verified by plotting a graph $F(\alpha)$, using expressions (13) – (15) with $-36^{\circ}\le\alpha\le 72^{\circ}$ in the Octave package. The continuous graph shown in Figure 1 is as follows.

The resulting CGA model (13) – (15) of the airship lift is an analytical function. It can be used in various mathematical equivalent transformations of the equations of the system model during synthesis.

5 Dynamic model of airship flight altitude control

To illustrate the possibility of solving problems of synthesis of control systems for objects, represented by CGA-models, we will construct MM for maneuvering an airship within a certain high-altitude echelon at its constant longitudinal velocity. In this case, we will assume that a change in flight altitude does not affect its lift. Formally, the airship has 6 degrees of freedom, but in our case, only 3 are important: longitudinal translational movement along the course, rotational movement in a plane $x-h$ and vertical movement in this plane caused by external influences (for example, upward or downward air currents) and a change in lift $F(\alpha)$. Let us assume that the constancy of the forward speed is provided by a special ACS.

Consequently, the sought model should describe: rotational motion in plane $x-h$, which depends on the influence of the elevators (u_h) and is determined by the acceleration

$$\ddot{\alpha}=x_2=M_u/J_D=LF_u/J_D=(Lk_F/J_D)u_h=k_hu_h \tag{16}$$

where J_d is moment of inertia of the airship about its horizontal transverse axis; $M_u=L\cdot F_u$ is moment created by an elevator located at a distance L from the center of mass and developing a pivot force F_u , and $x_2=\dot{x}_1$ is the angular rate of turn of the airship in the vertical plane; $k_h=L\cdot k_F/J_D$.

In addition, this model should describe the translational vertical motion caused by the action of the aerodynamic force $F(\alpha)$, the value of which is determined by expression (13). This force creates the vertical acceleration of the airship:

$$\ddot{h}=x_4=\left(F(\alpha)-\mu v_h|v_h|\right)/m_D=\left(x_1F(x_1)-\mu x_4|x_4|\right)/m_D \tag{17}$$

where $x_1 = \alpha$ is attack angle; μ is dynamic viscosity coefficient of air during vertical movement of the airship; $v = x_4$ is the speed of vertical movement, and $x_4 = \dot{x}_3$, $x_3 = h - h_g = \Delta h$, i.e. this is the deviation Δh of the current flight altitude h of the airship from the given value h_g ; m_d is the mass of the airship.

From the above expressions follows the following MM of the airship movements in the vertical plane

$$\dot{x} = O(x)x + d(x)u\,,\ h = x_3 + h_g\,, \tag{18}$$

where matrix $O(x)$ and vector $d(x)$ have the form

$$O(x) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \Gamma_1(x_1) & 0 & 0 & \Gamma_2(x_4) \end{bmatrix}, \qquad d(x) = \begin{bmatrix} 0 \\ k_h \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

where $\Gamma_1(\mathrm{x}_1) = F(\mathrm{x}_1)/m_D$ and $\Gamma_2(x_4) = -\left(\mu \cdot |x_4|\right)/m_D$.

6 Synthesis of the control law by the airship altitude

In accordance with the above synthesis method based on quasilinear MMs, first of all, the controllability of the obtained QLM is checked (18), (19). In this case, the determinant from condition (13) obviously does not vanish at all $x_1 \in [-36^{\circ} \div 72^{\circ}]$. Thus, QLM (18), (19) is completely controllable in the domain $x_1 \in [-36^{\circ} \div 72^{\circ}]$, $-\infty < x_i < \infty,\ i = 2, 3, 4$ and the solution to the synthesis problem exists. To determine it, an adjoint matrix is constructed to matrix $(\lambda E - O(x))$, which in this case has the form

$$\mathrm{adj}(\lambda E - O(x)) = \begin{bmatrix} \lambda^2(\lambda - \Gamma_2(x_4)) & \lambda(\lambda - \Gamma_2(x_4)) & 0 & 0 \\ 0 & \lambda^2(\lambda - \Gamma_2(x_4)) & 0 & 0 \\ \Gamma_1(x_1)\lambda & \Gamma_1(x_1) & \lambda^2(\lambda - \Gamma_2(x_4)) & \lambda^2 \\ \Gamma_1(x_1)\lambda^2 & \Gamma_1(x_1)\lambda & 0 & \lambda^3 \end{bmatrix}$$

Further, by formulas (7) and (8), taking into account equalities (18), (19) for n = 4, the polynomials are found:

$$O(\lambda, x) = \det(\lambda E - A(x)) = \lambda^4 - \Gamma_2(x_4)\lambda^3 \tag{20}$$

$$\left.\begin{aligned} V_1(\lambda, x) &= [1\ 0\ 0\ 0]\mathrm{adj}(\lambda E - O(x))d(x) = k_h\lambda^2 - \Gamma_2(x_4)k_h\lambda, \\ V_2(\lambda, x) &= [0\ 1\ 0\ 0]\mathrm{adj}(\lambda E - O(x))d(x) = k_h\lambda^3 - \Gamma_2(x_4)k_h\lambda^2, \\ V_3(\lambda, x) &= [0\ 0\ 1\ 0]\mathrm{adj}(\lambda E - O(x))d(x) = \Gamma_1(x_1)k_h, \\ V_4(\lambda, x) &= [0\ 0\ 0\ 1]\mathrm{adj}(\lambda E - O(x))d(x) = \Gamma_1(x_1)k_h\,s. \end{aligned}\right\} \tag{21}$$

Since we are synthesizing a 4th order system, in accordance with the above method, a stable 4th degree polynomial is formed.

$$S^*(\lambda)=\prod_{i=1}^{4}(\lambda-\lambda_i)=\lambda^4+r_3\lambda^3+r_2\lambda^2+r_1\lambda+r_0 \tag{22}$$

so that its roots satisfy the following conditions: $\lambda_i<0, \lambda_i\neq\lambda_j, i,j=\overline{1,4}$. Using polynomials (20) and (22), using formula (9), the coefficients of the polynomial are calculated as:

$$R(\lambda,x)=S^*(\lambda)-O(\lambda,x)=\left(r_3+\Gamma_2(x_4)\right)\lambda^3+r_2\lambda^2+r_1\lambda+r_0 \tag{23}$$

Based on polynomials $V_i(\lambda,x)$, $i=\overline{1,4}$ (21) and $R(\lambda,x)$ (23) according to formula (11), a system of algebraic equations is compiled, which in this case has the form:

$$\begin{bmatrix} 0 & 0 & \Gamma_1(x_1)k_h & 0 \\ -\Gamma_2(x_4)k_h & 0 & 0 & \Gamma_1(x_1)k_h \\ k_h & -\Gamma_2(x_4)k_h & 0 & 0 \\ 0 & k_h & 0 & 0 \end{bmatrix}\begin{bmatrix} k_1(x) \\ k_2(x) \\ k_3(x) \\ k_4(x) \end{bmatrix}=\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3+\Gamma_2(x_4) \end{bmatrix} \tag{24}$$

It is easy to verify that the determinant of the matrix of system (24) is not equal to zero, which is due to the controllability of the QLM (18), (19). Therefore, this system has a unique solution, substituting it into equality (2), we obtain the required memory:

$$u_h(x)=-k^T(x)\cdot x=-\left[\frac{r_2+r_3\Gamma_2(x_4)+\Gamma_2^2(x_4)}{k_h} \quad \frac{r_3+\Gamma_2(x_4)}{k_h} \quad \frac{r_0}{k_h\Gamma_1(x_1)} \quad \frac{r_1+r_2\Gamma_2(x_4)+r_3\Gamma_2^2(x_4)+\Gamma_2^3(x_4)}{k_h\Gamma_1(x_1)}\right]x. \tag{25}$$

Similarly, substituting control (25) into equation (18), we obtain the equation of the closed system in the form $\dot{x}=S(x)x$. In this case, the characteristic polynomial $S(\lambda,x)=\det(\lambda E-S(x))$ of the system matrix $S(x)$ coincides with the polynomial (22), i.e. is a Hurwitz polynomial. As shown in [10], under this condition, the equilibrium position $x=0$ of the resulting control system $\dot{x}=S(x)x$, $h=x_3+h_g$ is asymptotically stable in some finite region of its state space, including point $x=0$, that is, at $t\to\infty$ the state variables $x_i(t,x_0)\to 0$, $i=\overline{1,\ 4}$, $x_0\neq 0$ is the vector of initial conditions. Thus, the deviations of the flight altitude $h(t)$ of the airship caused by disturbances will be reduced to a given value of h_g under the action of control (27) at $t\to\infty$.

7 Computer study of the nonlinear control system by the airship flight altitude

In order to assess the actual stability and quality of the synthesized control system in the entire operating range of the supposed functioning of the control object, its digital modeling was carried out. The values of the roots of the characteristic polynomial $S^*(\lambda)$ (22) of the functional matrix $S(x)$ of the closed-loop system (18), (19), (25) on the nature of transient processes, two variants of their values were chosen.

The purpose of the control system in this case is to track the given law of change in the airship flight altitude and compensate for sharp deviations in flight altitude. Transient processes of changing the flight altitude $h(t)$ of the airship and control $u(t)$ when tracking a harmonically varying flight altitude task according to the law $h_g(t) = 1600 + 50\sin(t/250)$ м and a sharp deviation of altitude flights up to 300 meters are shown in figure 2 and figure 3 (continuous curves). Graphs of the given law of flight altitude change are shown in figure 2a and figure 3a dashed line. Transient processes in figure 2 correspond to the roots of the characteristic polynomial of the system matrix $S(x)$–equal to: –0.01; –0.03; – 0.05; – 0.07. Transient processes in figure 3 are obtained for the roots of the characteristic polynomial of the same matrix equal to: – 0.02; – 0.06; – 0.10; – 0.14.

As you can see, despite the sharp changes in the nature of the static characteristics of the airship, the changes in its flight altitude are smooth. This is the analytical nature of the CGA model of the airship altitude control channel. On the other hand, based on these figures, we can conclude that by changing the roots of the characteristic polynomial of the system matrix $S(x)$, one can change the speed of the control system. Indeed, if in the first case (figure 2) the duration of the transient process is about 300 seconds, then in the second case (figure 3), where the moduli of these roots are twice as large as in the first, the transient process lasts about 170 seconds, and the values control is significantly higher, which corresponds to the physics of the processes accompanying the controlled movement of the airship.

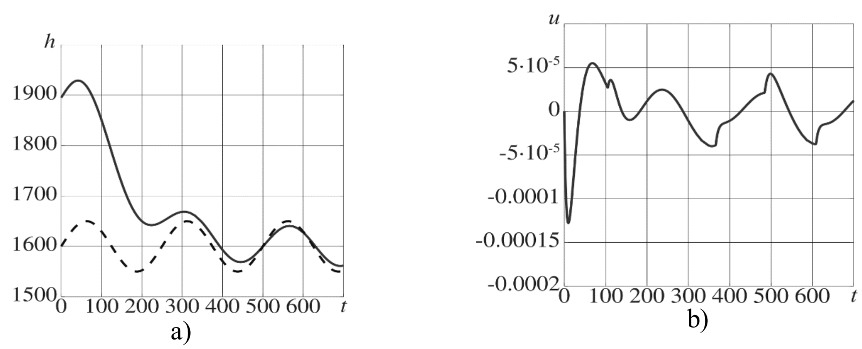

Fig. 2. Transient processes of a closed system in the first case: a) changes in flight altitude; b) control changes.

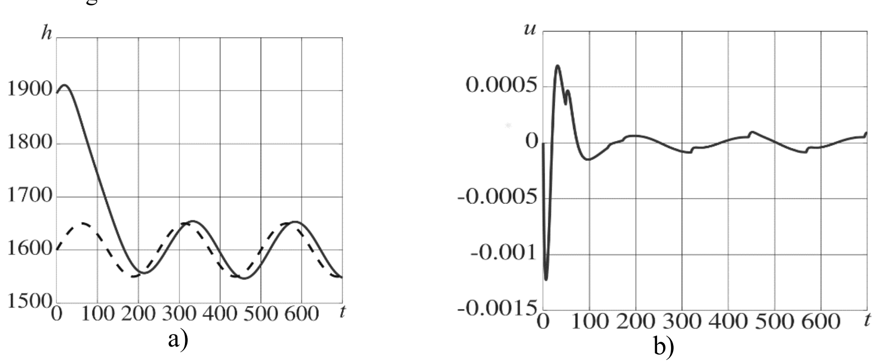

Fig. 3. Transient processes of a closed system in the second case: a) changes in flight altitude; b) control changes.

On the whole, the simulation results indicate the effectiveness of the synthesized nonlinear airship altitude control system, synthesized using quasilinear and CGA models of a substantially nonlinear control object.

8 Conclusion

Cut-Glue approximation method allows creating high-precision and high-quality analytical models of essentially nonlinear control objects based on experimental data of identification experiments, which significantly expands the scope of analytical methods for the synthesis of nonlinear ACS. The polynomial approximation of fragments of experimental data proposed in the article makes it quite easy to obtain quasi-linear models of complex nonlinear objects based on CGA-models, which, in turn, makes it possible to create effective nonlinear control laws for complex nonlinear objects. The results obtained can be used in the automation of production in the energy, automotive, aviation, chemical, agricultural and other industries.

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