Amplitude and phase relations in a two-circuit parametric circuit of ferroresonance nature

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Abstract. The article presented a mathematical analysis of a double-circuit parametric circuit of ferroresonance nature at the fundamental frequency, performed by the harmonic balance method. The adjustment, current-voltage, and load characteristics of the circuit are given. The possibility of using this circuit in voltage regulators with direct current output is proved. Parametric sources of secondary power supply, in particular voltage stabilizers of ferromagnetic and ferroresonance nature, are used in autonomous vehicles (for example, in spacecraft, some types of intelligent transport systems) and in renewable energy sources due to their ability to operate in heavy environments (high and low temperatures, radiation, strong magnetic or electric fields).

1 Introduction

New circuit solutions combined with advances in the field of magnetic materials make stable secondary power sources based on magnetic components, ferroresonance phenomena, and parametric resonance promising. As studies show [7-16], [18,21,22], the technical and economic parameters of parametric power supplies of low-and medium-power equipment are close to those of compensating stabilizers, and in some indicators (reliability, durability, temperature and time stability, resistance to heavy environments, low cost) exceed them. This allows us to conclude the relevance of the study of power supply circuits of parametric nature based on ferromagnetic components.

The purpose of this research is to obtain the amplitude-phase relations for a two-circuit parametric circuit of ferroresonance nature and to identify the stabilizing properties in circuit solutions based on this circuit.

The scheme is one of the models of the circuit of voltage regulator [9, 10] is shown in Fig.1, where S_1, S_2, square cross-sections, respectively, of left and right rods magnetic core; L_1, L_2,– the average length of the magnetic lines of the magnetic circuit; g1, g2 – active nonlinear conductivity of the windings of the coils NI1 and NI2; W_1, W_2 – the number of turns of nonlinear coils NI1 and NI2; C_1, C_2 – capacitance of capacitors; $i=Im*\sin(\omega t+\psi_i)$ – the instant value of the supply current; $u=Um*\sin(\omega t+\psi_u)$ – instant value of the supply voltage.

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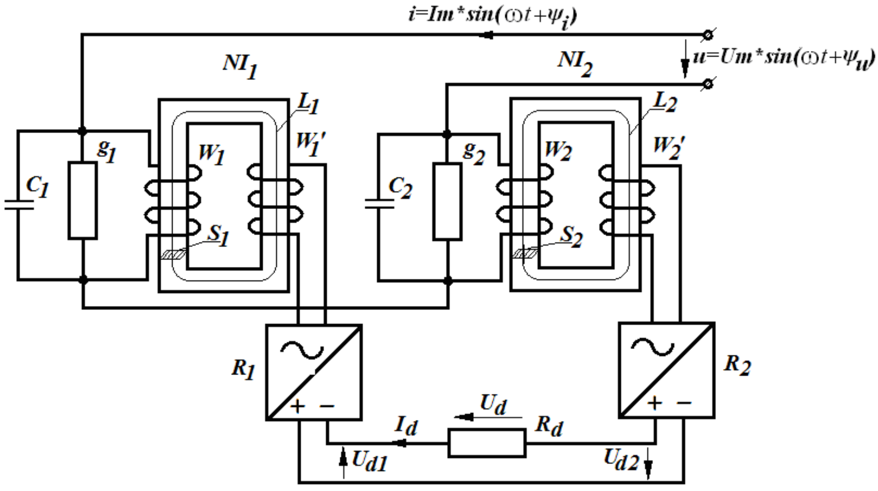

Fig.1. Model of the circuit of voltage regulator

The electrical state of the circuit can be described by a system of equations drawn up according to the first and second Kirchhoff laws for instantaneous values of electrical quantities

$$u = u_1 + u_2 \tag{1}$$

$$i = i_{C1} + i_{g1} + i_1 \tag{2}$$

$$i = i_{C2} + i_{g2} + i_2 \tag{3}$$

When using the approximation of the magnetization curve of the NI1-NIn cores by the expression $H = kB^9$, the use of which is justified in [1,4,11,12], as well as taking into account the known relations arising from the law of electromagnetic induction, the instantaneous values of the currents in the circuit elements can be found from the expressions

$$i_1 = \frac{k_1 l_1 b_1^9}{w_1} \tag{4}$$

$$i_2 = \frac{k_2 l_2 b_2^9}{w_2} \tag{5}$$

$$i_{C1} = w_1 C_1 s_1 \frac{d^2 b_1}{dt^2} \tag{6}$$

$$i_{C2} = w_2 C_2 s_2 \frac{d^2 b_2}{dt^2} \tag{7}$$

$$i_{g1} = w_1 g_1 s_1 \frac{db_1}{dt} \tag{8}$$

$$i_{g2} = w_2 g_2 s_2 \frac{db_2}{dt} \qquad (9)$$

Transform (1) and 3) and given (4)-(9), we get

$$w_1 C_1 s_1 \frac{d^2 b_1}{dt^2} + w_1 g_1 s_1 \frac{db_1}{dt} + \frac{k_1 l_1 b_1^9}{w_1} = w_2 C_2 s_2 \frac{d^2 b_2}{dt^2} + w_2 g_2 s_2 \frac{db_2}{dt} + \frac{k_2 l_2 b_2^9}{w_2} \qquad (10)$$

$$w_2 C_2 s_2 \frac{d^2 b_2}{dt^2} + w_2 g_2 s_2 \frac{db_2}{dt} + \frac{k_2 l_2 b_2^9}{w_2} = w_3 C_3 s_3 \frac{d^2 b_3}{dt^2} + w_3 g_3 s_3 \frac{db_3}{dt} + \frac{k_3 l_3 b_3^9}{w_3} \qquad (11)$$

2 Materials and Methods

The solutions for the instantaneous values of the inductions in (10) – (11) are assumed in the form $b_1 = B_{1m} \sin \omega t$; $b_2 = B_{2m} \sin(\omega t - \varphi_1)$, for which the expressions (10) and (11) are transformed by the harmonic balance method [16, 18, 23, 24]. Substitute the solution in (10) – (11) perform operations of differentiation and replacing the extent of harmonic functions by a sum of harmonics in the first degree and considering only members with a fundamental frequency, after transformations we get

$$\begin{aligned} & -w_1 C_1 s_1 \omega^2 B_{1m} \sin \omega t + w_1 g_1 s_1 \omega B_{1m} \cos \omega t + 0,5 \frac{k_1 l_1}{w_1} B_{1m}^9 \sin \omega t = \\ & = -w_2 C_2 s_2 \omega^2 B_{2m} \sin(\omega t - \varphi_1) + \\ & + w_2 g_2 s_2 \omega B_{2m} (\cos \omega t - \phi_1) + 0,5 \frac{k_2 l_2}{w_2} B_{2m}^9 \sin(\omega t - \phi_1) \end{aligned} \qquad (12)$$

Let's introduce the notation $\alpha_1 = w_1 C_1 s_1 \omega^2$; $\beta_1 = w_1 g_1 s_1 \omega$

$$\gamma_1 = \frac{0.5 k_1 l_1}{w_1} \qquad (13)$$

$$\alpha_2 = w_2 C_2 s_2 \omega^2 \ ; \ \beta_2 = w_2 g_2 s_2 \omega ; \ \gamma_2 = \frac{0.5 k_2 l_2}{w_2} \ ; \ \tau = \omega t$$

After replacing in (12) the sines and cosines of the sum of the arguments by the products of the sines and cosines, taking into account the notation (13), we get

$$\begin{aligned} & -\alpha_1 B_{1m} \sin \tau + \beta_1 B_{1m} \cos \tau + \gamma_1 B_{1m}^9 \sin \tau = -\alpha_2 B_{2m} \cos \phi_1 \sin \tau + \\ & + \alpha_2 B_{2m} \sin \phi_1 \cos \tau + \beta_2 B_{2m} \cos \phi_1 \cos \tau + \\ & + \beta_2 B_{2m} \sin \phi_1 \sin \tau + \gamma_2 B_{2m}^9 \cos \phi_1 \sin \tau - \gamma_2 B_{2m}^9 \sin \phi_1 \cos \tau \end{aligned} \qquad (14)$$

We transform (14) by the harmonic balance method. By equating the coefficients at $\sin\tau$ and $\cos\tau$ to the left and right of the equal sign, we obtain a system of algebraic equations

$$\begin{cases} (\gamma_2 B_{2m}^9 - \alpha_2 B_{2m})\cos\phi_1 + \beta_2 B_{2m}\sin\phi_1 = \gamma_1 B_{1m}^9 - \alpha_1 B_{1m} \\ -(\gamma_2 B_{2m}^9 - \alpha_2 B_{2m})\sin\phi_1 + \beta_2 B_{2m}\cos\phi_1 = \beta_1 B_{1m} \end{cases} \quad (15)$$

Squaring the expressions to the left and right of the equal sign in (15) and summing them, after the transformations, we get an expression describing the dependence between the amplitudes of the first harmonics of magnetic inductions in NI1 and NI2

$$(\gamma_2 B_{2m}^9 - \alpha_2 B_{2m})^2 + (\beta_2 B_{2m})^2 = (\gamma_1 B_{1m}^9 - \alpha_1 B_{1m})^2 + (\beta_1 B_{1m})^2 \quad (16)$$

Let's introduce the notation

$$\gamma_1 B_{1m}^9 - \alpha_1 B_{1m} = a_1;\ \gamma_2 B_{2m}^9 - \alpha_2 B_{2m} = a_2;$$
$$\beta_1 B_{1m} = d_1; \quad \beta_2 B_{2m} = d_2 \quad (17)$$

Given the notation (17), we multiply in (15) the upper expression by d_1, and the lower expression by a_1. Equating the left parts in the obtained expressions, after simple transformations, we find the value of the phase shift angle between the amplitudes of the first harmonics of the magnetic inductions in NI1 and NI2

$$\varphi_1 = arctg\left(\frac{a_1 d_2 - d_1 a_2}{d_1 d_2 + a_1 a_2}\right) \quad (18)$$

Let's consider the main characteristics of the device on the example of a physical model with parameters: C_2=20 μF, C_1=15 μF, g_1=g_2=0.0015 Om^{-1}, W_1=W_2=400 wind; S=0,00085 m^2, $L1$=$L2$=0.245m; $H=16.5\ *B^9$, magnetic core from steel E330 (3414). The dependencies B_{1m} and B_{2m} = f (I_m) are shown on the Figure 2.

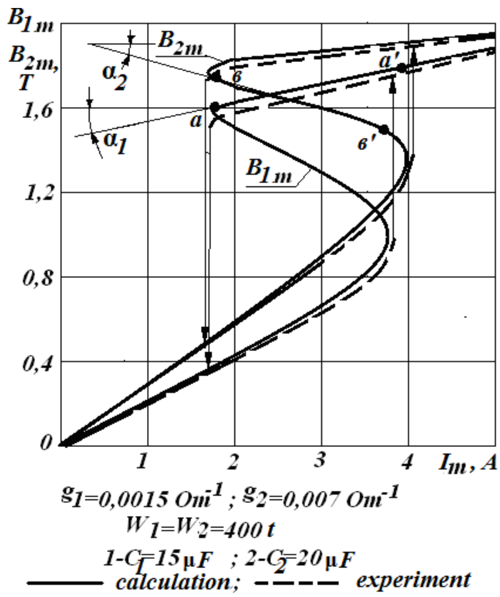

Fig. 2. The dependencies B_{1m} and B_{2m}

The figure shows that in some range of the input current when ferroresonance racing had occurred in both circuits, set the operation mode circuits, where the first FRK (NI1 coil and capacitor C_1) works in inductive mode on the section *a-a'* and the second FRK (NI2 coil and capacitor C_2) operates in capacitive mode on the section *b-b'*, the angles of these characteristics α_1 and α_2 with respect to the horizontal is approximately the same in meaning, but differ in sign (in the first case, the differential resistance is positive in the latter case, negative).

The amplitude of the first harmonic of the supply current (that is, the current I_m in the unbranched part of the circuit) can be found through the parameters of any of the FKK of this circuit. Given (2), (3), in accordance with Kirchhoff's first law, for the instantaneous values of the currents in the branches of FKK1 (the currents i_{C1}, i_{g1}, i_1, respectively, in the elements in the elements C_1, g_1, NI1), we will have

$$i = I_m \sin(\omega t + \psi_i) = i_{C1} + i_{g1} + i_1 = \frac{k_1 l_1 b_1^9}{w_1} + w_1 g_1 s_1 \frac{db_1}{dt} + w_1 C_1 s_1 \frac{d^2 b_1}{dt^2}$$

After performing the operations of differentiation and replacing the degree of harmonic functions with the sum of harmonics in the first degree, taking into account only the terms with the frequency of the main harmonic, after the transformations, we get

$$\begin{aligned} I_m \sin(\omega t + \psi_i) = i_{C1} + i_{g1} + i_1 = -w_1 C_1 s_1 \omega^2 B_{1m} \sin \omega t + \\ + w_1 g_1 s_1 \omega B_{1m} \cos \omega t + \frac{0.5 k_1 l_1}{w_1} B_{1m} \sin \omega t \end{aligned} \quad (19)$$

Replacing in (3-30) to the left of the equal sign the sine of the sum of the arguments by the product and taking into account (19), we have

$$I_m \cos\psi_i \sin\tau + I_m \sin\psi_i \cos\tau =$$
$$-\alpha_1 B_{1m} \sin\tau + \beta_1 B_{1m} \cos\tau + \gamma_1 B_{1m}^9 \sin\tau \qquad (20)$$

We transform (20) by the harmonic balance method, equating the coefficients for sinτ and cosτ to the left and right of the equal sign. Getting the system

$$\begin{cases} I_m \cos\psi_i = -\alpha_1 B_{1m} + \gamma_1 B_{1m}^9 \\ I_m \sin\psi_i = \beta_1 B_{1m} \end{cases} \qquad (21)$$

Squaring the expressions to the left and right of the equal sign in (21) and summing them, after the transformations, we get an expression describing the relationship between the amplitudes of the first harmonic of magnetic induction in NI1 and the current amplitude in the unbranched part of the circuit

$$I_m = \sqrt{(\gamma_1 B_{1m}^9 - \alpha_1 B_{1m})^2 + (\beta_1 B_{1m})^2} \qquad (22)$$

Dividing the lower expression in the system (21) by the upper one, we get the formula for determining the initial phase of the current

$$\psi_i = arctg\left(\frac{\beta_1 B_{1m}}{-\alpha_1 B_{1m} + \gamma_1 B_{1m}^9}\right) \qquad (23)$$

The voltage applied to the circuit can be found from the expression compiled according to the second Kirchhoff law for the instantaneous stress values for the general circuit of the generalizing ferromagnetic circuit. Taking into account the known relations arising from the law of electromagnetic induction, the instantaneous value of the applied voltage is determined by the expression

$$u = U_m \sin(\omega t + \psi_u) = u_1 + u_2 = w_1 s_1 \frac{db_1}{dt} + w_2 s_2 \frac{db_2}{dt}. \qquad (24)$$

After performing the differentiation operations, we get

$$U_m \sin(\omega t + \psi_u) = w_1 s_1 \omega B_{1m} \cos\omega t + w_2 s_2 \omega B_{2m} \cos(\omega t - \varphi_1). \qquad 25)$$

Let's introduce the notation

$$\upsilon_1 = w_1 s_1 \omega; \quad \upsilon_2 = w_2 s_2 \omega \qquad (26)$$

Replacing in (3-36) to the left and right of the equal sign the sine and cosine of the sum (difference) by products and taking into account the notation (26), we get

$$U_m \cos\psi_u \sin\tau + U_m \sin\psi_u \cos\tau = \upsilon_1 B_{1m} \cos\tau + \upsilon_2 B_{2m} \cos\tau \cos\varphi_1 +$$
$$+ \upsilon_2 B_{2m} \sin\tau \sin\varphi_1 + \qquad (27)$$

We transform (27) by the harmonic balance method, equating the coefficients for sinτ and cosτ to the left and right of the equal sign. Getting the system

$$\begin{cases} U_m \cos\psi_u = \upsilon_2 B_{2m} \sin\varphi_1 + \upsilon_3 B_{3m} \sin\varphi_2 \\ U_m \sin\psi_u = \upsilon_1 B_{1m} + \upsilon_2 B_{2m} \cos\varphi_1 + \upsilon_3 B_{3m} \cos\varphi_2 \end{cases}$$

after the transformation of which we find an expression for the amplitude of the voltage applied to the circuit as a function of the amplitudes of the magnetic inductions in the nonlinear inductances NI1-Nin

$$U_m = \sqrt{\begin{array}{l}(\upsilon_2 B_{2m} \sin\phi_1 + \upsilon_3 B_{3m} \sin\phi_2 +)^2 + \\ +(\upsilon_1 B_{1m} + \upsilon_2 B_{2m} \cos\phi_1 + \upsilon_3 B_{3m} \cos\phi_2 +)^2\end{array}} \tag{28}$$

as well as the initial phase of the voltage

$$\psi_u = arctg\left(\frac{\upsilon_1 B_{1m} + \upsilon_2 B_{2m} \cos\varphi_1 + \upsilon_3 B_{3m} \cos\varphi_2}{\upsilon_2 B_{2m} \sin\varphi_1 + \upsilon_3 B_{3m} \sin\varphi_2}\right) \tag{29}$$

From (23) and (29), the phase shift angle between the current vectors in the unbranched part of the circuit and the voltage applied to the circuit can be found

$$\varphi = \psi_u - \psi_i \tag{30}$$

3 Results and Discussion

Figure 3 shows the calculated and experimental current-voltage characteristics of the circuit under study.

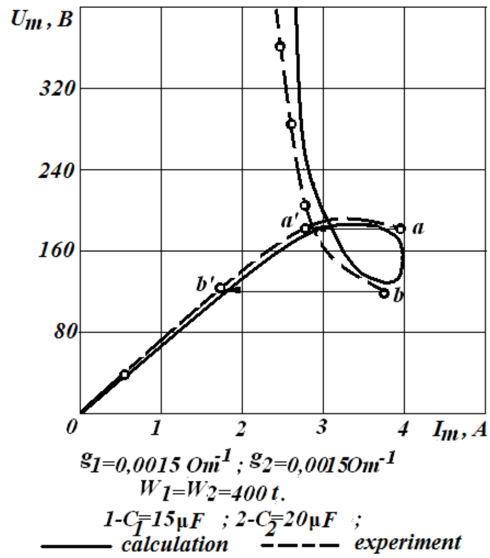

Fig. 3. Calculated and experimental current-voltage characteristics

It can be seen from Figure 3 that the range of current changes in which ferroresonance jumps occur (*a-a′* in the current growth mode and *b-b′* in the current reduction mode) corresponds to the jumps of electromagnetic inductions shown in Fig.2. Figure 3 also shows that the circuit in this range is powered in a mode close to the current source, which allows you to stabilize the voltage on both circuits according to the principle of operation of the Bouchereau circuit.

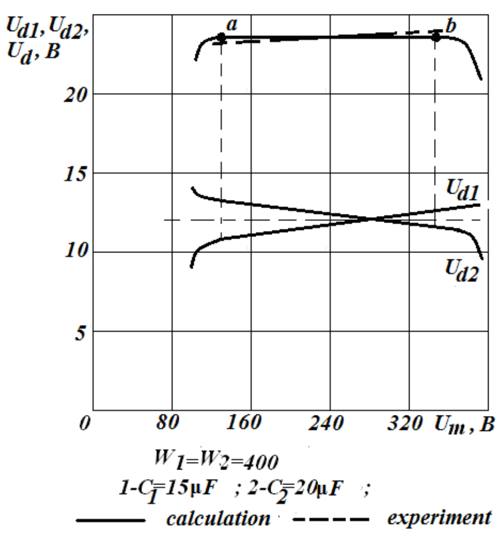

Fig. 4. shows the dependence of the rectified voltages on both circuits U_{d1} and U_{d2}, as well as the dependence of the load voltage U_d, equal to the sum of U_{d1} and U_{d2}, on the supply voltage U. From Fig. 4, it can be seen that in the range of changes in the input voltages *a-b*, which corresponds to a change in the voltage amplitudes from 100 to 340 V, the rectified voltage at the output of the stabilizer, determined by the expression $U_d = U_{d1} + U_{d2}$, practically does not change (the maximum deviation is about 1%).

4 Conclusions

1. As a result of a ferroresonance jump, the oscillatory circuits of a two-circuit circuit operate in two different modes– inductive and capacitive.
2. The circuit under study is powered in a mode close to the current source, which allows you to stabilize the voltage on the supplied circuit according to the principle of operation of the Bouchereau circuit.
3. Due to the power supply of the load from two circuits through series-connected rectifiers, the influence of phase shifts of voltages on circuits operating in inductive and capacitive modes on the value of the voltage on the load is eliminated, which improves the quality of stabilization

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