Development of a method for numerical modeling of the separation flow at the entrance to a rectangular exhaust channel with sharp edges

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Abstract. We have developed a method for mathematically simulating flow separation at the entrance to the exhaust channels of rectangular shape, using stationary discrete vortices in an ideal incompressible liquid in full spatial setting. When designing the discrete model, straight and curvilinear quadrangular vortex frames, straight and curvilinear horseshoe-shaped vortices were used. Using the developed computational procedure and computer program, the outline of vortex zone at the entry to the rectangular exhaust channel with sharp edges and the axial velocity of the air flow are determined. The results were compared with studies by different authors, as well as calculations by the methods of the theory of functions of a complex variables for the slotted exhausts and with results earlier obtained by us for calculation the square exhaust opening by means of discrete vortices in non-stationary and quasi-symmetric setting. The further direction of research will be related to the investigation of flows detached at the entrance to the rectangular exhaust hoods by the developed method, as well as by CFD methods. Then it is necessary to identify the regularities of the change of the local resistance coefficient at the entrance to the exhaust hood, shaped by the found outlines of the vortex zones.

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1 Introduction

To improve the efficiency of local exhaust ventilation, the most reliable way to capture pollutants, the most accurate information on the movement of air near the exhaust openings is needed. In particular, the definition of the boundaries of the detachable (vortex) zones is necessary for the determination of the boundaries for the shaping of the entry openings of local exhausts, which significantly reduces their aerodynamic resistance. The simulation of detachable flows entering the exhaust channels is usually done in a two-dimensional setting: flat or axially symmetric. Methods are used to calculate both an ideal incompressible liquid and a viscous medium. Previously, the detachable flows at the entrance to the slot and round exhaust openings in the hood form were studied in detail [1, 2]. The flow near square and rectangular exhaust ducts was investigated in a three-dimensional viscous medium model using CFD [3]. The flow separation from the sharp edge was shown, but the regularity of the change of the vortex zone outline was not considered**.**

2 Calculating method

The boundary of the exhaust channel is discretized by N vortex frames: rectangular and straight horseshoe shaped. The number of rectangular frames adjacent to a sharp edge is N_{sl}. At the Fig. 1 showed the first layer of square frames. The number of layers discretizing the side surface is K_{sl} (Fig. 2). The penultimate layer is replaced by straight horseshoe-shaped vortices (Fig. 3). The vortex elements on the side surface are numbered 1,2,…, N_1-1. The number of the vortex frame enclosing the active section is N_1. The exhaust section is discretized by the rectangular vortex frames with numbers N_1+1, N_1+2,…, N. Control (calculated) points are located in the center of the rectangular frames, the coordinates of which are calculated from the coordinates of the vertices a, b, c, d, as the arithmetic mean of the corresponding coordinates vertices. The calculated points for horseshoe-shaped vortices are located in the same place as for rectangular vortices (shown as crosses in Fig. 3). At the calculated boundary points, the normal components of the velocity are given, and at the solid boundaries sets the impermeability condition.

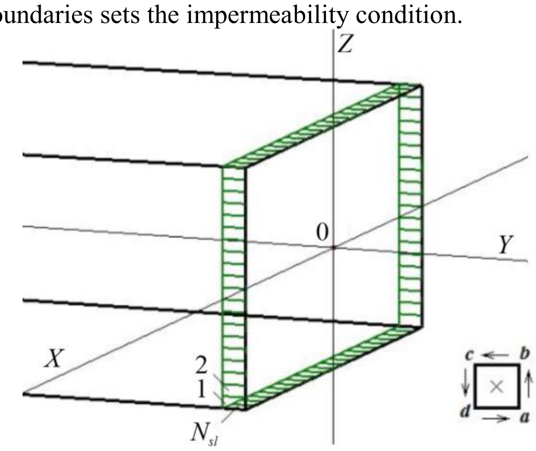

Fig. 1. First layer of square vortex frames

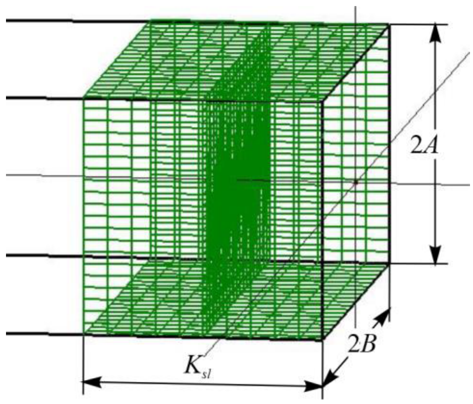

Fig.2. Ten layers (K_{sl} = 10) of square vortex frames along the side surface and exhaust section

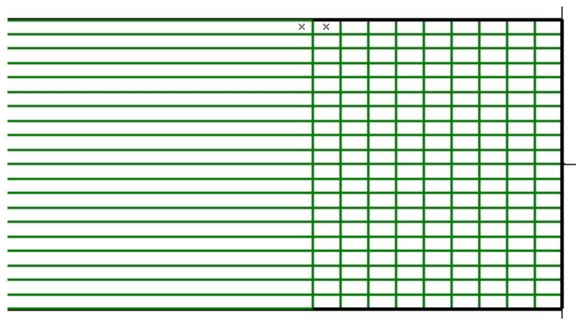

Fig.3. Transforming the last layer of the vortex frame into straight horseshoe-shaped vortices

The basis for the construction of the vortex system is a vortex straight segment. Velocity at point x along the unit direction $\boldsymbol{n}$ of the vortex segment of unit intensity outside the vortex core:

$$v_2(x) = \frac{1/r_1 + 1/r_2}{r_1 r_2 + \boldsymbol{r}_1 \cdot \boldsymbol{r}_2} \frac{[\boldsymbol{r}_1 \times \boldsymbol{r}_2] \cdot \boldsymbol{n}}{4\pi}, \quad (1)$$

where $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ is the radius of the vector of the end of the segment.
In the event of a vortex hitting the core (figure 4):

$$v_2(x,\boldsymbol{n}) = \frac{[\boldsymbol{r}_1 \times \boldsymbol{r}_2] \cdot \boldsymbol{n}(r_1 + r_2)\left(1 - \boldsymbol{r}_1 \cdot \boldsymbol{r}_2 / (r_1 r_2)\right)}{4\pi\left(r_1^2 r_2^2 - (\boldsymbol{r}_1 \cdot \boldsymbol{r}_2)^2 + r_h^2\left(r_1^2 + r_2^2 - 2\boldsymbol{r}_1 \cdot \boldsymbol{r}_2\right)\right)}. \quad (2)$$

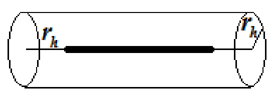

Fig.4. Vortex core

The effect on the control point of a quadrangular (in this case square) frame is expressed by the sum of four vortex segments in a given direction. Induced by a k-th straight horseshoe-shaped vortex (figure 5) of intensity $\Gamma = 1$ with rays pointing along $\boldsymbol{l}$, the velocity $\boldsymbol{v}$ at a point x along the direction $\boldsymbol{n}$ is calculated by means of a scalar product:

$$v_3(x,\boldsymbol{n},\boldsymbol{l},k) = \frac{\boldsymbol{n}}{4\pi} \cdot \sum_{j=1}^{2} \frac{\boldsymbol{r}_j \times \boldsymbol{l}}{\left|\boldsymbol{r}_j \times \boldsymbol{l}\right|^2} \left(1 + \frac{\boldsymbol{r}_j \cdot \boldsymbol{l}}{r_j}\right) + v_2(x,\boldsymbol{n}). \quad (3)$$

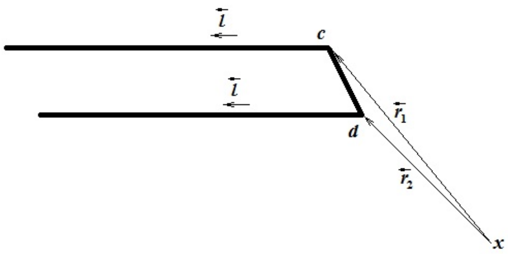

Fig.5. Influence of horseshoe-shaped vortex on a control point

The computational algorithms of the calculation method are as follows.
1. Forming G^{pq} matrix, p=1,2,... , N_1-1; q =1,2,... , N_1, effects on the calculation points of the connected vortex – square vortex frames and straight horseshoe-shaped vortices of unit intensity.
2. Matrix $G^{N_1 q}$ is formed, q =1,2,…, N_1, it effects on the calculation points t lying in the exhaust (active) section along the direction of the external normal $\boldsymbol{n}$ attached vortices with unit intensity:

$$G^{N_1 q} = \sum_{p=N_1+1}^{N} v(t,\boldsymbol{n},q)dS_p \,, \quad (4)$$

where $dS_p = h \cdot h$ is the area of the frame in the exhaust cross-section; $v(t, \boldsymbol{n}, q)$ is the function $v_3(t, \boldsymbol{n}, \boldsymbol{l}, q)$ or $v_4(t, \boldsymbol{n}, q)$ depending on the number q, under which the square or straight horseshoe-shaped vortices are numbered. The physical meaning of $v(t, \boldsymbol{n}, q) \cdot dS_p$ is the flow of air through the p-frame of the active section, which is induced by the q-th attached vortex.
The formed matrix G^{pq}, p=1,2,…, N_1; q =1,2,…, N_1, in the process of iterative definition of vortex free surface does not change.
3. Initial zero values are given for the number of vertices of free vortex break segments $\gamma_q^{new} = 0$, $\gamma_q^{old} = 0$, initial zero values of the circulation of free curvilinear frames $\gamma_r^{new} = 0$, $\gamma_r^{old} = 0$, their initial zero number. Then there is an iterative procedure for constructing a free vortex current surface going from the acute edge of the pipe.
3.1. Start of external iteration cycle.
3.2. The beginning of an internal iteration cycle which still holds the absolute value of the difference between the new and old circulation of the free vortex frame is greater than the given accuracy: $\left|\gamma_r^{new} - \gamma_r^{old}\right| > \varepsilon$. In this calculation ε = 10^{-5}.
3.3. Formation of free members of a system of linear algebraic equations to determine unknown intensities of vortex elements from boundary conditions for the normal velocity component:

$$v_p = v_p - \sum_{q=1}^{K_{sl}} v_{3\approx}(t_p,\boldsymbol{n},q)\gamma_q^{new} - v_r(t_p,\boldsymbol{n},N_{sv\_\min})\gamma_r^{new},$$
$$p = 1,2,...,N_1 - 1, \quad (5)$$

$$v_{N_1} = v_0 \cdot 4A \cdot B - \sum_{q=1}^{K_{sl}} \gamma_q^{new} \sum_{p=N_1+1}^{N} v_{3\approx}(t_p,\boldsymbol{n},q)dS_p -$$
$$-\gamma_r^{new} \sum_{p=N_1+1}^{N} v_r(t_p,\boldsymbol{n},N_{sv\_\min})dS_p, \quad (6)$$

where t_p is the calculation point - the midpoint of the p-th frame of the side surface of a rectangular channel, and $\boldsymbol{n}$ is the external normal at that calculation point.
3.4. Solving the system of linear algebraic equations for the unknown vortex frame circulations $\Gamma^1, \Gamma^2, ..., \Gamma^{N_1}$:

$$\sum_{q=1}^{N_1} G^{pq}\Gamma^q = v^p, \quad p = 1,2,...,N_1. \quad (7)$$

Obtaining new values of circulation for free vortex break segments and frames:

$$\gamma_q^{old} = \gamma_q^{new},\, q = 1,...,K_{sl}\,; \;\; \gamma_q^{new} = \Gamma^q,\, q = 1,...,K_{sl}\,. \quad (8)$$

If the external iteration cycle is repeated not for the first time, then $\gamma_r^{old} = \gamma_r^{new}$, $\gamma_r^{new} = \Gamma^{N_1} / N_{sv\_\min}$.
3.5. End of internal iteration cycle. The internal iteration cycle is repeated only once during the external iteration cycle for the first time.
3.6. Construction of free vortex break segments on a stream lines going from the sharp edges of the pipe with step h. The smallest number of vertices of break segments $N_{sv\_\min}^{new}$. The number of vertices on the vortex break segments are storing.
The number of vertices on free vortex break segments gets a new value: $N_{sv\_\min} = N_{sv\_\min}^{new}$.
3.7. End of external iteration cycle. If an internal iteration cycle is performed once, the process of determining a free vortex system is considered constructed. Then you can define the velocity field and build stream lines.

3 Results and discussion

An example of a free stream surface is given in Figure 6. Free curvilinear horseshoe-shaped vortices are arranged along the pipe from the sharp edge to the exhaust section. Closed curvilinear frames are arranged in the vertices of vortex break segments.

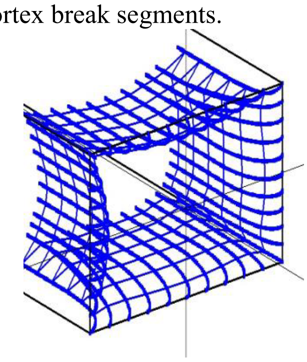

Fig.6. Free vortex system at inlet to square exhaust channel (A=B)

Using the developed computational procedure and computer program, the axial velocity of the air flow and the line of flow separation at the entrance to the rectangular exhaust channel with sharp edges are determined (Figure 7, 8). The free stream line was constructed from the middle of the sharp edge. The comparison of the results with the known and obtained with the help of CFD demonstrates the adequacy of the developed method.

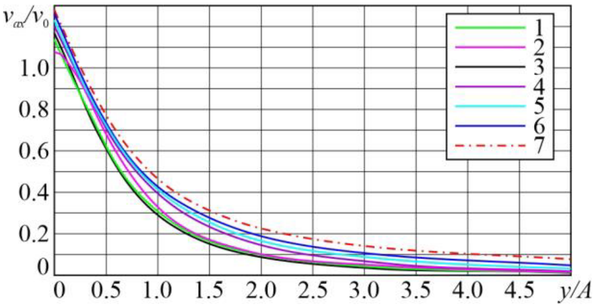

Fig.7. Variation of the dimensionless axial air velocity v_{ax}/v_0 near square (line 1, 2, 3), rectangular (line 4) and slot exhaust hoods (line 5).

The variation of the dimensionless axial velocity v_{ax}/v_0 (v_0 is the flow rate averaged exhaust velocity) of the square exhaust channel (Figure 7), constructed using the method of discrete vortices in stationary setting (Line 3) is well consistent with the Fletcher's formula calculation (line 2 [4]) and CFD calculating (line 1) using *k*-turbulence model with the STAR-CCM+ software. Note that on the cut of exhaust channel there is a slight increase in velocity relative to other methods. The highest velocity found by the N. E. Zhukovsky method is achieved for the slot exhaust (line 7), lines 4, 5 and 6 - changing of velocity near the rectangular exhaust at a factor of 2 to 1, 4 to 1 and 10 to 1 respectively. The last curve is closest to the changing of the axial velocity near the slot exhaust.

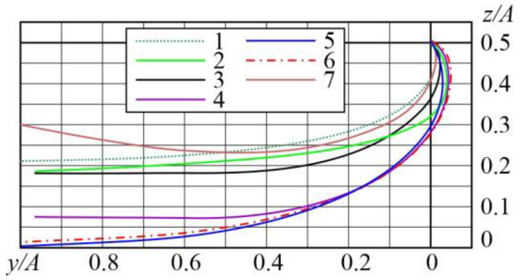

Fig.8. Separation (free) stream line at the entry of rectangular exhaust channel

The position of the free stream line 3 for the square exhaust channel is between the position of this stream line for the slot 5 and the circular exhaust channel 1 (figure 8). There is a marked difference in the line constructed according to the proposed formula which averages the position of the free stream line 2 for the quasi-symmetric problem in the non-stationary setting [5]. The free stream lines 4, 5 for a rectangular exhaust with a ratio of sides 4 to 1, 10 to 1 approaches line 6 and practically coincide with it. Line 7 shows the outline of the vortex zone found numerically with the STAR-CCM+ software for a square exhaust.

When moving away from the corner of the exhaust channel to the middle of the edge, the stream line is moving from the exhaust channel (figure 9). The thickness of the vortex zone is greatest in the plane passing through the midpoints of opposite edges and the axis of exhaust.

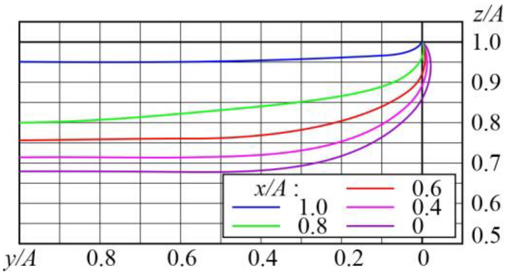

Fig.9. Free stream lines when the starting point of constructing is moving away from the corner of the square exhaust channel

The further direction of research will be related to the study of separated flows at the entrance of the rectangular exhaust hoods by the developed method, as well as by CFD methods. Then it is necessary to identify the regularities of the change of the local resistance coefficient at the entrance to the hoods, shaped along the found boundaries of the vortex zones. It is then necessary to study the rectangular-shaped exhaust hoods where two vortex zones can be formed. This method can also be used to study the formation of vortex zones in presence of cross drafts and ascending air flow.

Conclusion

A method has been developed for mathematical modeling of the separated flow at the inlet to the suction channels in a full spatial formulation. As an example, the calculation of the axial velocity and the boundaries of the vortex zone at the entrance to the square and rectangular suctions is made. The results obtained are adequate. The research was carried out under the grant of the Russian Science Foundation (project #23-49-00058) and NSFC (project #5221101677).

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