# Review of continuous adjoint method for inverse design of indoor thermal environment

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**Abstract.** The continuous adjoint method has been used in the indoor environment for more than ten years, but it has not been widely used up to now. This paper describes the current state of the continuous adjoint method and presents an overview of the continuous adjoint method for inverse regulation of the indoor environment from different aspects, including the inverse identification accuracy, inverse identification efficiency, inverse identification function, and globally optimal.

## 1 Introduction

Nearly 90% of the human lifetime is spent in the indoor environment[1], such as buildings, industrial plants, means of transportation, and so on. Therefore, it is very important to create a healthy and comfortable indoor environment. The indoor environment is usually regulated by heating, ventilating and air-conditioning (HVAC) systems and HVAC systems are equivalent to the respiratory system of the indoor environment. In the United States, the energy consumption of HVAC systems accounts for about 40% of the total building energy consumption[2]. Even with such high energy consumption, indoor environmental problems emerge one after another, including sick building syndrome[3], strong draft sensation[4], thermal discomfort, infectious virus airborne transmission through the HVAC system, and so on. So, how to create a healthy, comfortable, and energy-saving indoor environment needs to be solved urgently.

For such a scientific problem mentioned above, people usually want to optimize the input parameters of a system/model that follow a known physical law, so as to obtain the optimal output value. The traditional method is always a trial-and-error process. It constantly changes the input parameters according to the error between output value and target value and experience until the output value is close to or equal to the target value. For the problem of multi-parameter coupling, it is difficult to find the global optimal solution. In recent years, some researchers[5,6] tried to regulate the indoor environment through the inverse identification method. The inverse identification method has been favored by most researchers since it was introduced into the field of the indoor environment. The basic idea of the inverse identification method is to inversely determine the unknown optimal input parameters by continuously reducing the error between the output value and the target value. The inverse identification method can not only significantly reduce the workload but also identify better results.

The available inverse identification methods can be used to regulate the indoor environment includes the genetic algorithm (GA)[7], the proper orthogonal decomposition (POD) method[8], the artificial neural network[9], the continuous adjoint method[5,6], etc.. The principle of these methods is to seek the optimal design variables by minimizing the objective function, and they are classified as inverse identification methods by researchers in the field of the indoor environment. So, this study categorizes them as inverse identification methods. However, each method has its advantages and disadvantages in terms of inverse identification accuracy and efficiency. GA method can achieve global optimal, but the efficiency of this method is closely related to the number of design variables. When the design variables increase, the amount of calculation will increase exponentially. Note that, all these inverse identification methods adopted the Computational Fluid Dynamics (CFD) to predict the indoor airflow up to date. Since the CFD calculation time of each indoor environment case is very long, this method has recently been only used in the scenario with few design variables. Both the POD method and ANN method need to construct the functional relationship between the objective function and the design variables through a limited number of cases. Therefore, the possibility of finding the global optimal by the above two methods completely depends on the accuracy of the constructed functional relationship. Note that, the functional relationship of one case constructed by these two methods cannot be directly applied to other cases. For the continuous adjoint method, the amount of calculation is independent of the number of design variables which is the biggest advantage of that method, and the accuracy is very high, although this method may

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trap in the local optimal. Although the other three methods can also effectively inverse design indoor environments, this paper will focus on the continuous adjoint method for inversely identifying the optimal indoor environmental parameters.

The adjoint method has been known and used for nearly half a century. It was firstly proposed by lions[10] and used to solve the optimal control problem and the sensitivity analysis of the linear structural finite element model[11].

For the field of the indoor environment, Liu and Zhai[12] firstly used the adjoint method to inversely identify the contamination source's location in the building's indoor environment. Liu and Chen[13] further applied the adjoint method to inversely identify the air supply parameters for the optimal air distribution in enclosed spaces. In recent years, researchers tried to improve the performance of the continuous adjoint method from different aspects, including the inverse identification accuracy, inverse identification efficiency, inverse identification function, and global optimal, to further broaden its application range. Although great progress has been made, this method still has some shortcomings and limitations, such as local optimization, multi-objective problem, selection of optimization methods, etc. To facilitate readers to have a comprehensive understanding of this method and promote the continuous improvement of the continuous adjoint method, this paper summarizes the current progress of the continuous adjoint method, including the current hot research topics, application scenarios, and unsolved problems of this method.

## 2 Indoor environment design problems

To regulate an indoor environment that meets certain requirements, designers usually need to construct an appropriate objective function for quantitative evaluation. Commonly used evaluation indexes include predicted mean vote (PMV)[14], predicted dissatisfied percentage (PD) due to draft, the mean age of air $\tau$, energy consumption, and so on. For different specific problems, it may need to evaluate one or more of the above evaluation indexes simultaneously. To facilitate the method description, the definition of the objective function as shown in Eq. (1) is used as an example, the objective function $J(\xi)$ is a function of air velocity $\mathbf{V}$, air temperature T, or other parameters.

$$J(\xi)=\int_{\Theta} f(\mathbf{V},T,\cdots)d\Theta \qquad (1)$$

where $\xi$ is the design variables; $\Theta$ represents the design domain which refers to the region concerned by the people.

## 3 Continuous adjoint method

Since the relationship between design variables and objective function as shown in Eq. (1) cannot be expressed explicitly, it is very difficult to obtain $dJ/d\xi_n$ directly. The continuous adjoint method transforms the constrained optimization problem with m variables and k constraints into an unconstrained optimization problem with (m + k) variables by introducing a set of adjoint variables (adjoint pressure $p_a$, adjoint velocity $\mathbf{V}_a$, adjoint temperature $T_a$ …). The adjoint variables represent the change of the extreme value of the objective function when the constraint conditions change. The new objective function is the augmented objective function L:

$$L = J + \int_{\Omega}\left(p_a, \mathbf{V_a}, T_a\right)\cdot \mathbf{N}d\Omega \qquad (2)$$

$\mathbf{N}$ equal zero when the N-S equations were fully convergent solved, solving $dJ/d\xi$ can be equivalent to solving $dL/d\xi$. With the solution of N-S equations[15] and the corresponding continuous adjoint equations, $dL/d\xi$ can be easily obtained by the following formula:

$$\frac{dL}{d\xi} = \frac{\partial J}{\partial \xi} + \int_{\Omega}\left(p_a, \mathbf{V_a}, T_a\right)\frac{\partial \mathbf{N}}{\partial \xi}d\Omega \qquad (3)$$

### Inverse identification process

The inverse identification process starts with the initial boundary conditions. With the given boundary conditions, the indoor airflow field can be predicted by solving the N-S equations until convergence is achieved. Then the method checks whether the objective function satisfies the convergence criteria? If so, the inverse identification process was ended. Otherwise, the corresponding adjoint equations are solved until it converges completely. With the solutions of the primal N-S equations and the adjoint equations, the gradient of the objective function over the design variables can be obtained. Finally, the optimization method updated the design variables. The above inverse identification process continues until the objective function reaches the convergence criteria. The convergence criteria used in the inverse identification process are usually set according to the accuracy requirements of practical problems. Note that, when the continuous adjoint method combined with the topology method or FEM used to inversely identify the optimal inlets' location, number, size, shape, the topology method does not require remeshing and FEM requires remeshing[16,17].
Development of the continuous adjoint method

### 3.1 Inverse identification accuracy

To obtain the accurate gradient of the objective function over the design variables and improve the accuracy of inverse identification, Zhao and Chen[18] derived the adjoint equations of the Reynolds-averaged Navier-Stokes (RANS) equations closed with the turbulence model[19]. Since the RNG k-ε turbulence model is widely used to predict the airflow in the indoor environment, Zhao and Chen[18] developed the adjoint RNG k-ε turbulence model and their results indicated that the 'frozen turbulence' assumption indeed produces large errors. Note that, the method used to derive the adjoint turbulence model of all the other eddy-viscosity models is similar, readers can develop other adjoint turbulence models using the same method.

### 3.2 Inverse identification accuracy

The available methods used to improve the efficiency of inverse identification were through different strategies, including numerical solution method (multi-zone model[20], FFD[21], 'frozen turbulence' assumption, zero-equation eddy-viscosity models, sub-iteration method, coarse grid, etc.), optimization method (adaptive step size, etc.), computer technology (graphics processing unit (GPU), etc.) and so on.

### 3.3 Inverse identification function

Although researchers try to improve the accuracy and efficiency of the continuous adjoint method from different aspects, this method cannot meet the required identification functions, such as determination of the airborne contaminant source location[22], the optimal location, number, size, shape of air supply inlet[23], real air supply inlets[24], heat source[25] and so on. To meet these special requirements, some researchers try to introduce new methods to expand the scope of application of the continuous adjoint method.

### 3.4 Global optimal

Even if we obtain the accurate gradient using the continuous adjoint method, the objective function may fall into the local optimal and the local optimal value will not always satisfy our requirements. Currently, the available methods that can be used to inverse regulate the indoor environment includes GA, POD method, ANN method, and the continuous adjoint method, and so on. However, each method has its advantages and disadvantages mentioned in the introduction section. Wei et al. [26] tried to combine the GA method, POD method, and the continuous adjoint method to inverse identify the optimal aircraft cabin indoor environment. Their results indicated that the integrated method could find the global optimal. However, the bottleneck of this method is that the amount of numerical calculation is very large. Therefore, novel methods combined with the continuous adjoint method which can find the global optimum need to be developed.

## 4 Conclusions

This paper overviews the research progress of the continuous adjoint method used to inversely regulate the indoor environment. The main research topics include inverse identification accuracy, inverse identification efficiency, inverse identification function, global optimal.

The accuracy of inverse identification can be greatly improved by the established adjoint turbulence model instead of using the 'frozen turbulence' assumption.

The available methods to improve the inverse identification efficiency include multi-zone model, FFD, coarse grid, GPU, adaptive step size, and so on. Not that, each method has its own advantages and disadvantages, and a single method has limited ability to improve the efficiency of inverse identification, so the coupling of multiple methods would be the best choice.

The available probability-based adjoint method could efficiently determine the source location of airborne contaminants. Both the finite element method and topology method are suitable for inversely identifying the inlets' location, number, size. However, the inverse identification of real inlet is not limited to the momentum method, it can be any potential method, such as the methods in Srebric and Chen[27].

The combination of the GA method, POD method, and the adjoint method could find the global optimal, but its result was at the cost of a huge amount of calculation. More efficient methods need to be explored.

The methods mentioned in this paper are not the sole solution to a specific problem, and better solutions need to be further explored in the future.

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