A mathematical model of the room temperature dynamic response in multi-zone buildings

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Abstract. There are many factors that affect the change of room temperature, especially outdoor environmental factors, which makes it difficult to establish a model to describe the dynamic changes of the real time room temperature. In response to this problem, this study proposes a RC network two-node wall structure method to describe and analyze the heat conduction process of the building envelope. With this method, we first divide building envelope such as walls and floors into two-story structures, and then use energy balance theory to establish the energy balance equation of each node. Finally, we obtained the dynamic response equations of room temperature and humidity in each room of the whole building. This method simplifies the complex heat transfer process of the building envelope to a simple mathematical problem. The experimental validation indicated the accuracy of the proposed model.

1 Introduction

The room temperature depends on the building envelope, indoor heat source, outdoor environment, and the operation of the variable air volume (VAV) terminal. In order to control the room temperature precisely in multi-zone buildings, we first need to establish a mathematical model to predict the dynamic response of room temperature accurately [1-2]. Some researchers have already established multi-zone mathematical models to simulate room temperature changes under different working conditions in multi-zone buildings, by using the resistance-capacitance (RC) network method to simplify the building envelope model [3-5]. For example, Jin et al. [6] used the heat capacity and thermal resistance network method to divide the wall and other building envelope into one-layer structure and established a multi-zone mathematical model of air-conditioning in a room with partition walls. Although the model can accurately calculate the energy consumption of the room and ensure indoor comfort, it does not consider the coupling of heat transfer between floors and indoor air flow in multi-zone buildings, which causes some deviation in simulating the dynamic response of room temperature. Therefore, it is necessary to establish a more accurate multi-zone building mathematical model to reduce the dynamic response deviation of room temperature in the simulation process.

2 The multi-zone building mathematical model

2.1 Energy and humidity balance equations

The RC network two-node wall structure method can transform the complex heat transfer process of the building envelope into a simple mathematical problem and provide a new idea to establish a mathematical model of the room temperature dynamic response.

2.1.1 Typical zones of the room

As shown in Fig. 1, the room is separated into three typical zones, including the air-supply zone, the work zone, and air-return zone. Air in each zone is assumed to be fully mixed.

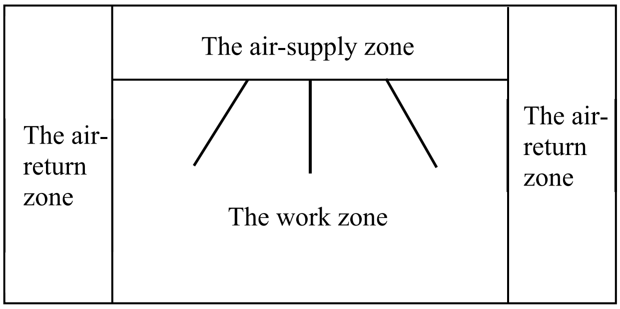

Fig. 1. Schematic for the air-conditioning room

2.1.2 Nodes of the wall

The wall and floor of a room are divided into two layers as shown in Fig.2. The heat capacity and resistance of each layer of walls and floor are concentrated on the

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central node and the corresponding nodes of the indoor and outdoor contact surfaces.

The thickness between the outer surface of the wall and the second node of the wall is 1/4 of the thickness of the wall, and the thickness between the two nodes of the wall is 1/2 of the thickness of the wall. The thickness between the surfaces is 1/4 of the wall thickness.

We made the following assumptions to simplify the model:

(1) The air properties are assumed to be the same for each zone.

(2) The envelop enclosure wall is divided into two layers and the heat transfer is one-dimensional.

(3) The energy transmitted from solar radiation and indoor heat source to the wall through radiation are constants.

We ignored the transmission of indoor humidity through the building envelope, or the radiative heat transfer between indoor walls and objects.

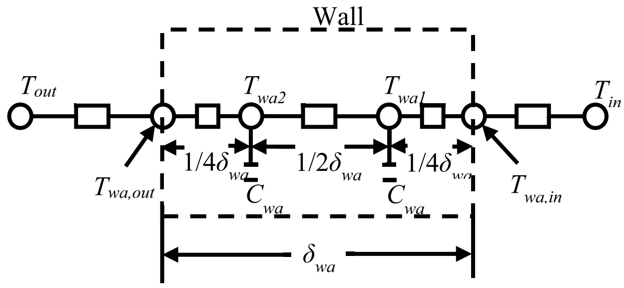

Fig. 2. Simulation method of the RC network

2.1.3 Energy balance equations of the wall

We analyzed three heat transfer processes of building envelope: (1) convection heat transfer process between outdoor air and exterior surface of building envelope; (2) heat conduction process from outer surface to inner surface of building envelope; (3) convection heat transfer process between indoor air and interior surface of building envelope. We also considered heat transfer from indoor heat sources, solar radiation and windows.

For single node of room wall, the equation is shown as follows:

The energy change at a single node = Incoming energy in unit time - Outgoing energy in unit time (1)

Therefore, the energy balance equation of the outer surface of the wall, the second node of the wall, the first node of the wall and the inner surface of the wall can be obtained by Eq. (1), as follows:

$$\frac{\lambda_{wa}}{\frac{1}{4}\delta_{wa}}A_{wa}\left(T_{wa2}\text{-}T_{wa,out}\right)+A_{wa}K_{wa,out}\left(T_{out}\text{-}T_{wa,out}\right)+Q_{sunfu}=0 \quad (2)$$

$$\frac{1}{2}\rho_{wa}\delta_{wa}A_{wa}C_{wa}\frac{dT_{wa2}}{dt}=\frac{\lambda_{wa}}{\frac{1}{2}\delta_{wa}}A_{wa}\left(T_{wa1}\text{-}T_{wa2}\right)+\frac{\lambda_{wa}}{\frac{1}{4}\delta_{wa}}A_{wa}\left(T_{wa,out}\text{-}T_{wa2}\right) \quad (3)$$

$$\frac{1}{2}\rho_{wa}\delta_{wa}A_{wa}C_{wa}\frac{dT_{wa1}}{dt}=\frac{\lambda_{wa}}{\frac{1}{2}\delta_{wa}}A_{wa}\left(T_{wa2}\text{-}T_{wa1}\right)+\frac{\lambda_{wa}}{\frac{1}{4}\delta_{wa}}A_{wa}\left(T_{wa,in}\text{-}T_{wa1}\right) \quad (4)$$

$$\frac{\lambda_{wa}}{\frac{1}{4}\delta_{wa}}A_{wa}\left(T_{wa1}\text{-}T_{wa,in}\right)+A_{wa}K_{wa,in}\left(T_{inr}\text{-}T_{wa,in}\right)+Q_{sunen}+Q_{inner}=0 \quad (5)$$

where A_{wa} is the wall area; λ_{wa} is the thermal conductivity of the wall; δ_{wa} is wall thickness; $K_{wa,out}$ is the heat transfer coefficient of the outer wall and outdoor air; $K_{wa,in}$ is the heat transfer coefficient of the inner wall and indoor air; T_{out} is the outdoor temperature; $T_{wa,out}$ is the temperature of the outer wall surface; T_{wa1} is the temperature of the first node in the wall; T_{wa2} is the temperature of the second node in the wall; $T_{wa,in}$ is the temperature of the wall internal surface; ρ_{wa} is the wall density; C_{wa} is the heat capacity of the wall; T_{inr} is the temperature of the air-return zone; Q_{sunfu} is the solar radiation energy absorbed by the external surface of the wall; Q_{sunen} is the solar radiation energy hitting the inner surface of the wall; Q_{inner} is the energy transferred to the inner surface of the wall by radiation for the indoor heat source.

$$\frac{\lambda_{fl}}{\frac{1}{4}\delta_{fl}}A_{fl}\left(T_{fl2}\text{-}T_{fl,out}\right)+A_{fl}K_{fl,out}\left(T_{fl,do}\text{-}T_{fl,out}\right)=0 \quad (6)$$

$$\frac{1}{2}\rho_{fl}\delta_{fl}A_{fl}C_{fl}\frac{dT_{fl2}}{dt}=\frac{\lambda_{fl}}{\frac{1}{2}\delta_{fl}}A_{fl}\left(T_{fl1}\text{-}T_{fl2}\right)+\frac{\lambda_{fl}}{\frac{1}{4}\delta_{fl}}A_{fl}\left(T_{fl,out}\text{-}T_{fl2}\right) \quad (7)$$

$$\frac{1}{2}\rho_{fl}\delta_{fl}A_{fl}C_{fl}\frac{dT_{fl1}}{dt}=\frac{\lambda_{fl}}{\frac{1}{2}\delta_{fl}}A_{fl}\left(T_{fl2}\text{-}T_{fl1}\right)+\frac{\lambda_{fl}}{\frac{1}{4}\delta_{fl}}A_{fl}\left(T_{fl,in}\text{-}T_{fl1}\right) \quad (8)$$

$$\frac{\lambda_{fl}}{\frac{1}{4}\delta_{fl}}A_{fl}\left(T_{fl1}\text{-}T_{fl,in}\right)+A_{fl}K_{fl,in}\left(T_{inr}\text{-}T_{fl,in}\right)+Q_{sunen}+Q_{inner}=0 \quad (9)$$

where λ_{fl} is the thermal conductivity of the floor; δ_{fl} is the floor thickness; $T_{fl,do}$ is the downstairs temperature; $K_{fl,out}$ is the heat transfer coefficient between the outer surface of the floor and downstairs air; $K_{fl,in}$ is the heat transfer coefficient between the inner surface of floor and indoor air; $T_{fl,out}$ is the downstairs roof temperature; T_{fl1} is the first node temperature of the floor; T_{fl2} is the second node temperature of the floor; $T_{fl,in}$ is the interior surface temperature of the floor; A_{fl} is the floor area; ρ_{fl} is the floor density; C_{fl} is the floor heat capacity.

2.1.4 Energy balance equations of indoor air

Energy balance equations of the air-supply zone, air-return zone and work zone can be obtained by Eq. (1).

The temperature equation in the air-supply zone is:

$$\rho_a V_{ins} C_a \frac{dT_{ins}}{dt}=K_{wa,in}A_{wa,s}\left(T_{wa,in}\text{-}T_{ins}\right)+K_{fl,in}A_{fl,s}\left(T_{fl,in}\text{-}T_{ins}\right)+G_{sa}C_a\left(T_{sa}\text{-}T_{ins}\right) \quad (10)$$

where T_{ins} is the temperature of the air-supply zone; T_{sa} is the supply air temperature; G_{sa} is the mass flow of the supply air; V_{ins} is the volume of the air-supply zone; ρ_a is the air density; C_a is the air heat capacity; $A_{wa,s}$ is the contact area between the air-supply zone and wall; $A_{fl,s}$ is the contact area between the air-supply zone and floor.

The temperature equation in the work zone is:

$$\left(\rho_a V_{inw} C_a+C_f\right)\frac{dT_{inw}}{dt}=K_{wa,in}A_{wa,w}\left(T_{wa,in}\text{-}T_{inw}\right)+K_{fl,in}A_{fl,w}\left(T_{fl,in}\text{-}T_{inw}\right)+K_{la}A_{la}\left(T_{la}\text{-}T_{inw}\right)+G_{sa}C_a\left(T_{sa}\text{-}T_{inw}\right)+K_{win}A_{win}\left(T_{out}\text{-}T_{inw}\right)+\sum_i \rho_a V_{adj,i} C_a R\left(T_{adj,i}\text{-}T_{inw}\right) \quad (11)$$

where T_{inw} is the temperature of the work zone; $A_{wa,w}$ is the contact area between the work zone and wall; K_{win} is the heat transfer coefficient of the window and outdoor air; A_{win} is the area of the window; $A_{fl,w}$ is the contact area between the work zone and wall; T_{la} is the average temperature of the indoor heat source; G_{sa} is the mass flow of the supply air; V_{inw} is the volume of the work zone; C_f is the indoor furniture heat capacity; R is the number of air changes; $V_{adj,i}$ is the volume of the adjacent room i; $T_{adj,i,}$ is the temperature of the adjacent room i; K_{la} is the heat transfer coefficient of the heat source.

The temperature equation in the air-return zone is:

$$\rho_a V_{inr} C_a \frac{dT_{inr}}{dt} = K_{wa,in} A_{wa,r}\left(T_{wa,in} - T_{inr}\right) + G_{sa} C_a\left(T_{inw} - T_{inr}\right) + G_{sa} C_a\left(T_{ins} - T_{inr}\right) \quad (12)$$

where V_{inr} is the volume of the air-return zone; $A_{wa,r}$ is the contact area between the air-return zone and wall.

The humidity equation in the air-supply zone is:

$$\rho_a V_{ins} \frac{dM_{ins}}{dt} = G_{sa} M_{sa} - G_{sa,sw} M_{ins} \quad (13)$$

where G_{sa} is the mass flow of the supply air; M_{sa} is the humidity of the supply air M_{ins} is the air humidity of air-supply zone; $G_{sa,sw}$ is the air mass flow from the air-supply zone to the work zone.

The humidity equation in the work zone is:

$$\rho_a V_{inw} \frac{dM_{inw}}{dt} = G_{sa1} M_{ins} - G_{sa,wr} M_{inw} + \sum_i \rho_a V_{adj,i} C_a R\left(M_{adj,i} - M_{inw}\right) \quad (14)$$

where $G_{sa,wr}$ is the air mass flow from the work zone to the air-return zone; M_{inw} is the humidity of the work zone; $M_{adj,i}$ is the humidity of the adjacent room i.

The humidity equation in the air-return zone is:

$$\rho_a V_{inr} \frac{dM_{inr}}{dt} = G_{sa,wr} M_{inw} - G_{sa1} M_{inw} + \sum_i \rho_a V_{adj,i} R\left(M_{adj,i} - M_{inw}\right) \quad (15)$$

where M_{inr} is the air humidity of the air-return zone; G_{sa1} is the mass flow of the return air.

2.2 Single-zone model

First, we established a single-zone model. The matrix form of the first-order differential equation can be obtained by collating Eq.(2) - (15) as follows:

$$\mathbf{C}\frac{d\mathbf{X}}{dt} = \mathbf{AX} + \mathbf{BU} \quad (16)$$

where **A**, **B**, **C** are correlation coefficients; **X** is the system variable vector; **U** is the disturbance vector.

Multiplying both sides of the equation with $\mathbf{C}^{-}$, we have:

$$\frac{d\mathbf{X}}{dt} = \mathbf{C}^{-}\mathbf{AX} + \mathbf{C}^{-}\mathbf{BU} \quad (17)$$

Eq.(17) is integrated over time, and the solution of **X** is:

$$\mathbf{X}(t) = e^{\mathbf{C}^{-}\mathbf{AX}}\mathbf{X}(0) + e^{\mathbf{C}^{-}\mathbf{AX}} \int_0^t e^{-e\mathbf{C}^{-}\mathbf{A}\tau}\mathbf{C}^{-}\mathbf{BU}(\tau)d\tau \quad (18)$$

2.3 Multi-zone model

Second, we extended the single-zone mathematical model to the multi-zone mathematical model. We established the coupling model of temperature and humidity in multi-zone building, and got the dynamic response equation of the temperature and humidity in each room. After that, we obtained the temperature and humidity of each room in the whole building. Assuming a building has n rooms numbered from 1 to n, then the multi-zone model shall be:

$$\begin{bmatrix} \mathbf{C}_1 & 0 & 0 & 0 \\ 0 & \mathbf{C}_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{C}_n \end{bmatrix} \begin{bmatrix} \frac{d\mathbf{X}_1}{dt} \\ \frac{d\mathbf{X}_2}{dt} \\ \vdots \\ \frac{d\mathbf{X}_n}{dt} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & 0 & 0 & 0 \\ 0 & \mathbf{A}_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A}_n \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_n \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 & 0 & 0 & 0 \\ 0 & \mathbf{B}_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{B}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \vdots \\ \mathbf{U}_n \end{bmatrix} \quad (19)$$

The solution to Eq. (19) is the same to Eq. (16), then $\mathbf{X}_n$ as:

$$\mathbf{X}_n(t) = e^{\mathbf{C}_n^{-}\mathbf{A}_n\mathbf{X}_n}\mathbf{X}_n(0) + e^{\mathbf{C}_n^{-}\mathbf{A}_n\mathbf{X}_n} \int_0^t e^{-e\mathbf{C}_n^{-}\mathbf{A}_n\tau}\mathbf{C}_n^{-}\mathbf{B}_n\mathbf{U}_n(\tau)\, d\tau \quad (20)$$

3 Experimental Validation

We did experimental validation for the proposed multi-zone model. Instruments for measuring the building envelope and indoor/outdoor environmental parameters were calibrated before experiment. These parameters are shown in Table 1.

Table 1. The building envelope and environmental parameters

Variables	Parameters	Variables	Parameters
Long	5.9 m	ρ_a	1.34 kg/m³
Wide	3.9 m	G_{sa}	0.05 kg/s
High	3.3 m	K_{win}	3 W/(m²·°C)
A_{win}	2 m²	C_f	10000J/(kg·°C)
V	75.933 m³	$T_{wa,out}$	19.5°C
ρ_{wa}	2800 kg/m³	$K_{wa,out}$	12 W/(m²·°C)
δ_{wa}	0.2 m	$K_{fl,out}$	6 W/(m²·°C)
A_{wa}	12.87 m²	ρ_{fl}	2500 kg/m³
C_{wa}	1000 J/(kg·°C)	δ_{fl}	0.2 m
λ_{wa}	1.6 W/(m·°C)	C_{fl}	950 J/(kg·°C)
$K_{wa,in}$	5 W/(m²·°C)	$T_{fl,out}$	18.5°C
C_a	1000 J/(kg·°C)	F	3386.11 W/m²
T_{sa}	10°C	M_{sa}	10.6 g/(kg dry air)
λ_{fl}	12.8 W/(m·°C)	M_{in}	11.5 g/(kg dry air)
T_{in}	15°C	Q_{sunfu}	0.4 KJ
$K_{fl,in}$	5 W/(m²·°C)	Q_{suner}	0.5 KJ
A_{fl}	23.01 m²	Q_{inner}	0.35 KJ
T_{out}	23°C		

3.1 Humidity simulation

The humidity of the room is simulated in the MATLAB software according to the building envelope and indoor and outdoor environmental parameters. As shown in Fig. 3, although the room air humidity ratio is constantly changing, the variation is small, with values between 10.6 g/(kg dry air) and 11.5 g/(kg dry air). The humidity ratio tends to be stable and close to M_{sa}.

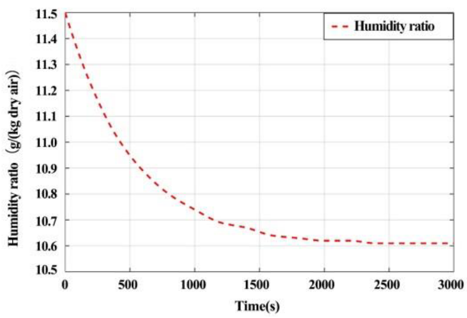

Fig. 3. The humidity response curve of room

3.2 Temperature simulation

The temperature of the room was simulated in the MATLAB software according to the building envelope and indoor and outdoor environmental parameters. The comparison for simulated and measured results are shown in Fig.4-6. These figures indicated that our proposed multi-zone model using the RC network two-node wall structure method can accurately simulate the dynamic change of room temperature.

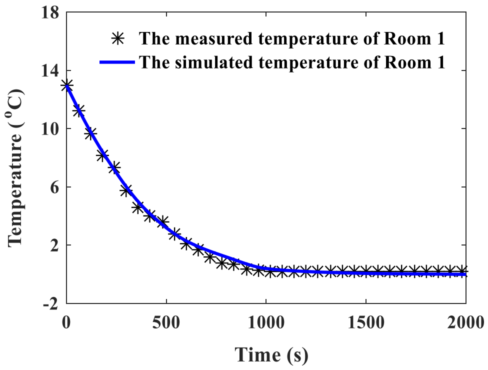

Fig. 4. Temperature response curves of Room 1

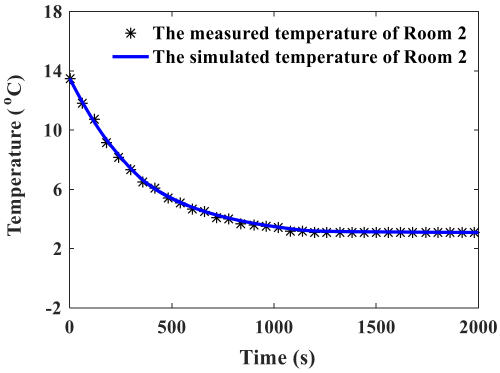

Fig. 5. Temperature response curves of Room 2

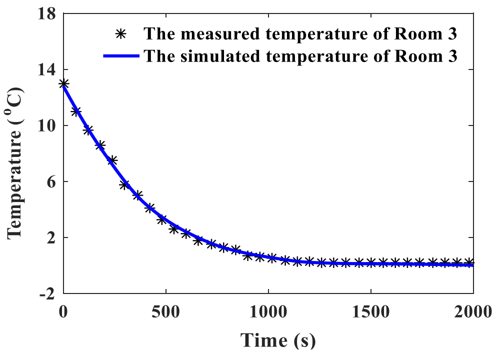

Fig. 6. Temperature response curves of Room 3

4 Conclusions

This research proposes a RC network two-node wall structure method to describe the dynamic changes of the real time room temperature. This method first divides building envelope such as walls and floors into two-story structures, and then uses energy balance theory to establish the energy balance equation of each node. Finally, the dynamic response equations of room temperature and humidity in each room of the whole building are obtained. Experimental validation indicated that the proposed method can accurately simulate the dynamic change of room temperature and humidity inside multi-zone buildings.

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