

Intelligent decision support in the optimization of irrigation systems in agriculture

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Abstract. The issues of determining the optimal values of the regulatory parameters of irrigation systems engaged in the cultivation of agricultural crops are considered. Following the requirements of a market economy, the main emphasis is placed on taking into account two types of criteria: maximizing the yield of agricultural crops and minimizing monetary costs. The proposed method for solving the multi-criteria optimization problem is based on the combination of the minimax criterion and the medium-step convolution, which makes it possible to scalarize the vector optimality criterion with access to smooth optimization methods. Concerning the case of priority uncertainty according to particular optimality criteria, an intelligent algorithm is proposed based on the approximation of the preference function of the decision-maker by the fuzzy Mamdani model. The multi-criteria optimization of the irrigation system used for growing cotton results differ favorably from the average values. The one hectare yield in the republic- increased by 2%, monetary costs - reduced by 4.5%. It could be concluded that the developed methodology makes it possible to bypass the computational difficulties that arise when solving problems of multi-criteria optimization of irrigation systems engaged in the cultivation of agricultural crops and to obtain real results in conditions of certainty and uncertainty goals.

1 Introduction

One of the effective ways to solve the problems of rational use of water and land resources in water and agriculture, increasing crop yields is using system analysis, mathematical modeling, and optimization in planning, preparing, and managing the technological processes of irrigation systems.

A feature of setting the optimization problem in a market economy about irrigation systems of agriculture is multi-criteria, since along with technological (yield), it involves the use of economic optimality criteria (cash costs for irrigation, mineral fertilizers, etc.). The vector optimality criterion gives rise to two problems in practice [1-5].

First, the existing vector optimality criteria scalarization methods often lead to non-smooth functions. Conventional numerical optimization methods under these conditions

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turn out to be ineffective due to the emerging «jamming» effect. Overcoming this problem is possible based on smoothing the scalar optimality criterion and is in the plane of solving ill-posed problems by regularization methods [6- 8]. The practical implementation of these methods encounters several difficulties due to the need for additional functional analysis and relatively high computational costs.

Secondly, most vector optimality criteria scalarization methods require setting priorities (weight coefficients) for each particular optimality criteria. In practice, setting priorities is not a trivial task and leads to uncertainty. The solution to the problem of «uncertainty of priorities» is possible based on the introduction of a preference function (PF) of a decision maker (DM), followed by its approximation by fuzzy or neural network approximation methods [9-12].

Thus, further progress in solving problems of multi-criteria optimization in the preparation and control of technological processes of irrigation systems of agriculture provides for the creation of effective methods and algorithms that combine traditional numerical optimization methods, regularization methods for ill-posed problems, and intelligent decision support.

2 Methods

Multicriteria optimization of irrigation systems used for cultivation of agricultural crops under conditions of certainty. The problem statement for optimizing the irrigation system has the form

$$y_1 \rightarrow \max_{x \in \Omega_v}; y_2 \rightarrow \min_{x \in \Omega_v}, \quad (1)$$

In (1) y_1 is the yield

$y_2 = c_1x_1 + c_2x_3 + c_3x_4 + c_4x_5 + c_5x_6 + c_6 + c_7 + c_8 + c_9 + c_{10} + c_{11}$ is money spendings; x_1 is irrigation rate per integrated hectare (thousand m³/hectare); x_3 is costs of nitrogen fertilizers (Ton/hectare); x_4 is costs of phosphate fertilizers (Ton/hectare); x_5 is labor costs (man-days/hectare); x_6 is average consumption of seeds (kg/hectare); c_1 is cost of irrigation (sum/thousand m³); c_2 is cost of nitrogen fertilizers (sum/t); c_3 is cost of phosphate fertilizers (sum/Ton); c_4 is the average wage rate (sum/person-day); c_5 is average cost of seeds (sum/kg); c_6 is average cost of land reclamation works (sum/hectare); c_7 is the average cost of sowing (sum/hectare); c_8 is average cost of crop protection from pests and diseases (sum/hectare); c_9 is costs for services (sum/hectare); c_{10} is expenses for the needs of management (sum/hectare); c_{11} is other expenses for social contributions (sum/hectare); Ω_v is the set of feasible solutions.

The set of feasible solutions is defined as:

$$\Omega_v = Y \cap X = \{Y \in R^k | y_1 \geq t_{1task}; y_2 \leq t_{2task}; X \in R^n | x_{jmin} \leq x_j \leq x_{jmax}; j = \overline{1, n}\} \quad (2)$$

Applying the combined convolution method, we transform problem (1) as follows [2, 15].

Multiplying both parts of the criterion constraint $y_1 \geq t_{1task}$ in expression (2) by -1, we obtain constraints of the form $-y_1 \leq t_{1task}$ and a particular criterion $-y_1 \rightarrow \min_{x \in \Omega_v}$. Let us

introduce estimates of the degree of fulfillment of constraints for each of the output parameters of the form

$$z_i(x) = \alpha_i[(t_i - y_i)/\delta_i - 1] \geq 0; \quad i = \overline{1,2}; \quad \alpha_i \geq 0; \quad \sum_{i=1}^2 \alpha_i = 1, \quad (3)$$

where δ_i is the estimate of the scattering of the i -th output parameter, which is set based on practical considerations or is determined using the method of statistical tests; α_i is weight coefficients that determine the relative importance of particular criteria y_i ; $i = \overline{1,2}$.

Applying the maximin convolution, we obtain a scalar optimality criterion

$$F(x) = \min_{i=1,2} z_i(x) \rightarrow \max_{x \in D}, \quad (4)$$

where D is a set in which direct restrictions on the variable parameters with the help of an appropriate substitution, for example, $x_j = x_{jmax} + (x_{jmin} - x_{jmax}) \cdot \sin^2(x'_j)$, converted to functional; x'_j ; $j = \overline{1,n}$ - new independent variables.

Let's smooth the criterion (4) using the exponential function and the power-mean convolution

$$F(x) = \sum_{i=1}^2 \phi_i^\gamma(x) \rightarrow \min_{x \in D}; \quad \gamma = 1, 2, \dots, \quad (5)$$

In (5) $\phi_i(x) \equiv \exp[-z_i(x)]$, γ is parameter introduced to control convergence in the vicinity of the optimum point.

The final optimization problem (1) will take the following form

$$F(x) = \sum_{i=1}^2 \exp[-\gamma \cdot z_i(x)] \rightarrow \min_{x \in D}; \quad \gamma = 1, 2, \dots, \quad (6)$$

In (6), $F(x)$ is the modified optimality criterion.

As applied to additive regression, for the smoothness of the modified criterion, it is necessary that the partial derivatives be continuous [15]. When the condition of smoothness of the modified criterion is satisfied, as applied to problem (6), the simplest $f'_i(x_i, a)_{x_i}$; $i = \overline{1,n}$ smooth optimization algorithms can be applied in practice.

Intelligent Decision Support in the Problem of Multicriteria Optimization of Irrigation Systems under Uncertainty of Priorities. When solving problem (6), the values may not be known in advance, which leads to the α_i , $i = \overline{1,2}$ uncertainty of priorities. In this case, the general statement of the problem of multi-criteria optimization of irrigation systems is formulated as follows. A vector function is given, whose components are particular optimality criteria and defined on the set of alternatives of the vector of variable parameters $\Psi(x, a) = (y_1(x, a), y_2(x, a))$ $y_1(x, a)$ $y_2(x, a)$ Ω_X X . It is necessary to find such a solution on the set, Ω_X would minimize all components of the vector function $\Psi(x, a)$.

For each fixed vector, the combined convolution method reduces the solution of problem (6) to the solution of a single criterion optimization problem of the form:

$$A = (\alpha_1, \alpha_2)$$

$$\min_{x \in D} F(x, A) = F(x^*, A) \quad (7)$$

If the solution of problem (7) is unique for each $A \in D_A = \{A \mid \alpha_i \leq 0, \sum_{i=1}^2 \alpha_i = 1\}$, then this means that each of the admissible vectors A corresponds to a single vector x^* and corresponding values of partial optimality criteria $y_1(x^*, a)$, $y_2(x^*, a)$. Based on this, you can build some PF DM $\zeta(A)$, defined on the set $D_A : \zeta : A \rightarrow R$.

Then the problem of multi-criteriative optimization is reduced to the choice of $A^* \in D_A$, such that $\max_{A \in D_A} \zeta(A) = \zeta(A^*)$.

We will assume that ζ it is a linguistic variable that takes a certain number of finite values, for example, $e=5$: «Very bad», «Bad», ..., «Very Well». Let us ζ^0 denote the kernel of the fuzzy variable ζ and introduce the following correspondence: the value ζ «Very bad» corresponds to $\zeta^0 = 1$, the value ζ «Bad» corresponds to $\zeta^0 = 2$, the value ζ «Average» corresponds to $\zeta^0 = 3$, the value ζ «Well» corresponds to $\zeta^0 = 4$, and the value ζ «Very Well» corresponds to $\zeta^0 = 5$.

This, the problem of multi-criteria optimization is reduced to finding a vector $A^* \in D_A$, that provides the maximum of the discrete function $\zeta(A)$:

$$\zeta(A^*) = \max_{A \in D_A} \zeta(A), \quad (8)$$

those, to the approximation of the PF DM.

The general scheme for solving such a problem is iterative and has several stages [1, 13].

At the first stage, n vectors A_1, A_2, \dots, A_m are randomly generated. The order of the following actions is as follows.

A one-criteria problem is solved:

$$F(x^*, A) = \min_{x \in D} F(x, A_l), l = \overline{1, m} \quad (9)$$

The obtained values are $x_l^*; l = \overline{1, m}; y_i(x_l^*); i = \overline{1, 2}$.

The obtained values are evaluated $y_i(x_l^*); i = \overline{1, 2}; l = \overline{1, m}$ and the values of the preference function $z(A_l); l = \overline{1, m}$ are introduced.

In the *second step*, based on the values A_1, A_2, \dots, A_m and estimates $z(A_l); l = \overline{1, m}$, the following actions are performed.

1) A function $\tilde{\zeta}_1(A)$ is constructed, approximating $\zeta(A)$ in the vicinity of points A_1, A_2, \dots, A_m ;

2) A single-criteria problem is solved

$$\max_{A \in D_A} \tilde{\zeta}_1(A) = \tilde{\zeta}(A_1^*); \quad (10)$$

3) A single-criteria problem $\min_{x \in OD} F(z, A_1^*) = F(x^*, A_1^*)$ is solved;

4) The found values $x^*; y_i(x^*); i = \overline{1, 2}$ are displayed;

5) The obtained values $y_i(x^*); i = \overline{1, 2}$ are evaluated, and the value of the preference function $\zeta(A_1^*)$ is entered.

In the *third step*, based on the available values of $A_1, A_2, \dots, A_m, A_1^*$ and the corresponding estimates of the preference function $\zeta(A_1), \zeta(A_2), \dots, \zeta(A_k), \zeta(A_1^*)$, an approximation of the function $\zeta(A)$ in the vicinity of points $A_1, A_2, \dots, A_m, A_1^*$ is performed, as a result of which the function $\tilde{\zeta}_2(A)$ is constructed. Further, the procedure continues according to the scheme of the second stage until the DM decides to stop the calculations. At each iteration, a «rollback» is allowed to change the previously introduced estimates of its PF DM.

The approximation of PF DM $\zeta(A)$, given indistinctly as a linguistic variable, can be carried out by fuzzy models, neural and neuro-fuzzy networks [9-12].

3 Results of practical application

The above optimization technique has been applied to the irrigation system used for cotton cultivation.

The software implementation of optimization algorithms was carried out in the MATLAB 2015 environment on a computer with an Intel(R) Core (TM) i5-9400 CPU @ 2.90 GHz and 8.00 GB of RAM.

The vector of input parameters of the technological process included: irrigation rates per complex hectare x_1 (thousand m^3 /hectare), cash costs x_2 (thousand sum/hectare), costs of nitrogen fertilizers x_3 (Ton/hectare), costs of phosphate fertilizers x_4 (Ton/hectare), labor costs x_5 (person-days/hectare).

On the output parameters y_1 (yield) and y_2 (monetary costs), the input parameters $x_1 \div x_5$ of the technological process of cotton irrigation, restrictions were imposed, constituting a set of permissible solutions

$W_v = Y I X = \{Y O R^2 | y_1 \text{ i } 37 \text{ Centner / hectare};$
 $y_2 \text{ J } 86000 \text{ thousand sum / hectare};$
 $X O R^5 | 6 \text{ thousand } m^3 / \text{hectare J } x_1 \text{ J } 10 \text{ thousand } m^3 / \text{hectare};$
 $80000 \text{ thousand sum / hectare J } x_2 \text{ J } 86000 \text{ thousand sum / hectare};$
 $0,2 \text{ T / hectare J } x_3 \text{ J } 0,25 \text{ T / hectare}; 0,15 \text{ T / Ha J } x_4 \text{ J } 0,175 \text{ T / hectare};$
 $x_5 = 720 \text{ person - days / hectare}\}.$

The equalization of the mathematical model and yield has the form of an additive regression equation

$$y_1 = \frac{a_1 x_1}{(a_2 + x_1)^2} + a_3 x_2 + a_4 x_3 + a_5 x_4 + a_6 x_5, \tag{11}$$

where $a_1 = 420.06$; $a_2 = 6.49$; $a_3 = 0.00000002$; $a_4 = 0.075$; $a_5 = 0.075$; $a_6 = 0.01505$.

The optimization problem under conditions of certainty was to maximize parameter y_1 and minimize parameter y_2 . A set of variable parameters make up parameters x_1, x_3 , and x_4 .

The scattering estimates of the output parameter values were selected as follows: $d_1 = 5, d_2 = 150000$. The static optimization problem in proposition (6) was solved by the method of coordinate descent for different sets of values $\alpha_i; i = 1, 2$. The results of the optimization are summarized in Table 1. Bold indicates a situation in which the functional limits on the output parameters are violated.

Table 1. Results of software implementation of the optimization model.

№	α_1	α_2	$y_i; i = \overline{1,2}$	F	x_1	x_3	x_4	It's time decisions, sec.
1	0	1	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.7964	6.0000	0.2000	0.1500	19.22
2	0.1	0.9	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.8151	6.0000	0.2000	0.1500	19.16
3	0.2	0.8	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.8342	6.0000	0.2000	0.1500	19.22
4	0.3	0.7	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.8537	6.0000	0.2000	0.1500	19.22
5	0.4	0.6	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.8737	6.0000	0.2000	0.1500	19.16
6	0.5	0.5	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.8941	6.0000	0.2000	0.1500	19.22
7	0.6	0.4	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.9150	6.0000	0.2000	0.1500	19.22
8	0.7	0.3	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.9364	6.0000	0.2000	0.1500	19.22
9	0.8	0.2	$y(1) = 38.5605502$ $y(2) = 79023222.5$	1.9582	6.0000	0.2000	0.1500	19.16
10	0.9	0.1	$y(1) = 38.5744754$ $y(2) = 79026271.1$	1.9805	6.1596	0.2000	0.1500	19.22
11	1	0	$y(1) = 38.7631635$ $y(2) = 87627619.4$	2.0032	6.4920	0.2500	0.1750	19.11

Optimization in the face of uncertainty of priorities was carried out as follows.
 The formation of function values ζ was carried out based on the rules given in Table 2.

Table 2. Function value generation rules.

$N\grave{o}$	F	ζ
1	$F \leq 1.8$	<i>Very Well</i>
2	$1.8 < F \leq 1.86$	<i>Well</i>
3	$1.8 < F \leq 1.92$	<i>Average</i>
4	$1.92 < F \leq 2$	<i>Bad</i>
5	$F > 2$	<i>Very bad</i>

When solving the problem (6), the method of coordinate descent was used, and the method of the golden ratio was used to solve the problem (10). The intermediate points of the function $z(A) = z(a_1, a_2)$ were determined using a cubic spline. The approximation of the function ζ was carried out using a fuzzy Mamdani model, which was implemented using the **Fuzzy Logic Toolbox** MATLAB 2015 extension [16, 17].

The semi-complete fuzzy output system (see Fig. 1) has two inputs (weight1, weight2), a Mamdani fuzzy output mechanism, and one output (function). The input variables are a_1, a_2 the weighting coefficients of the particular criteria of optimality y_1 and y_2 ; the output variable corresponds to the function ζ .

The input and output variables correspond to the coziness of the membership function, which were given as a symmetrical Gaussian function.

The input variables correspond to three types of membership functions: small, middle, big, which correspond to a small, medium, and large value of the weighting coefficients a_1 and a_2 . The output variable corresponds to five types of accessory functions, which have been assigned names - VB (*Very bad*), B (*Bad*), A (*Average*), W (*Well*), VW (*Very Well*).

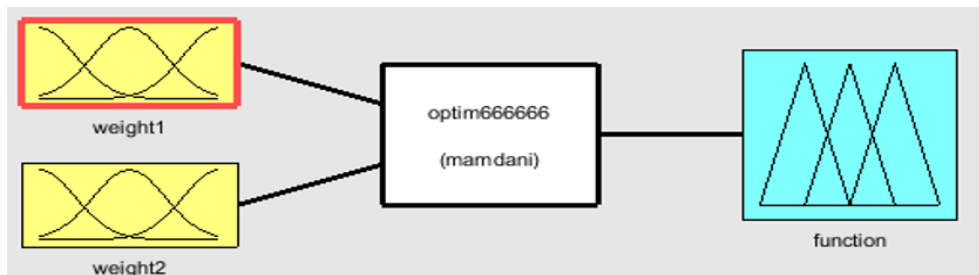


Fig. 1. Fuzzy Output System

The set of rules that specify the relationship between the input and output variable is of the form

- IF weight1 = small AND weight2 = big THEN function = Very Well;*
- IF weight1 = small AND weight2 = middle THEN function = Well;*
- IF weight1 = middle AND weight2 = middle THEN function = Average;*
- IF weight1 = big AND weight2 = middle THEN function = Bad;*
- IF weight1 = big AND weight2 = small THEN function = Very bad.*

Fine-tuning the fuzzy output model at each step of the problem solution (10) was implemented using the *fmincon* function of the extension **Optimization Toolbox MATLAB 2015** [18-20].

In Fig. 2, an illustration of a fuzzy model is given Mamdani, obtained after tuning in the next step of solving the problem (10).

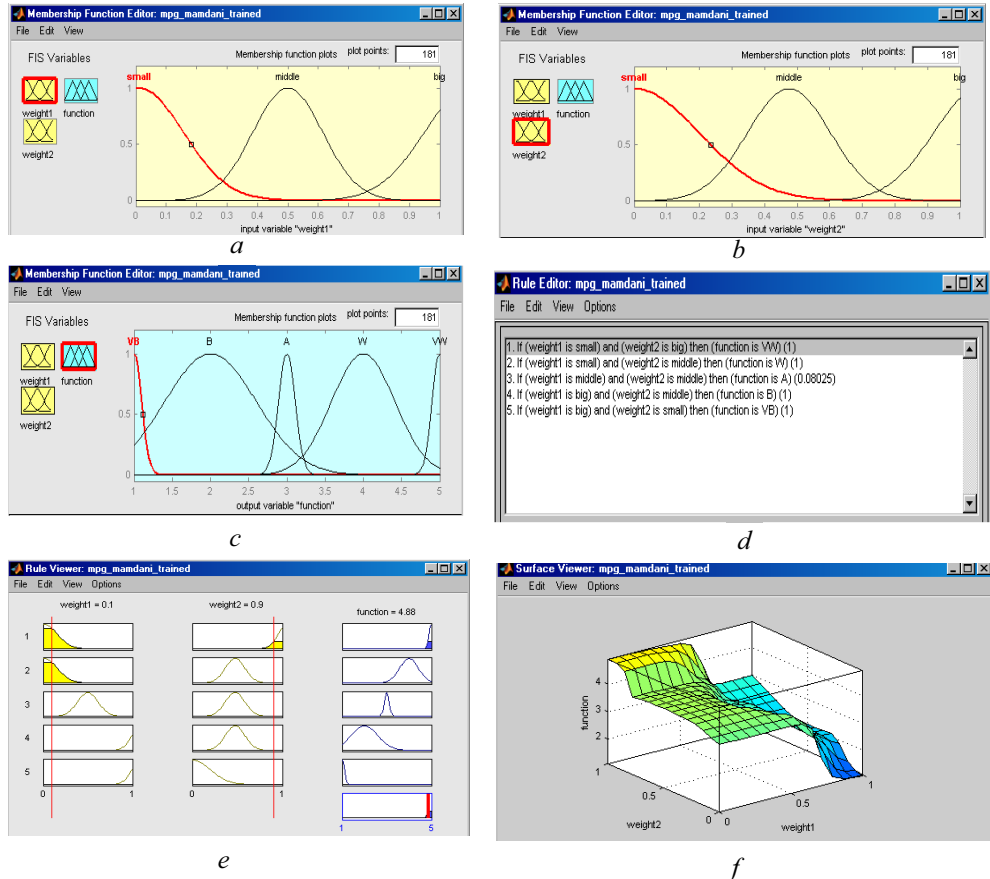


Fig. 2. Fuzzy Mamdani model after fine-tuning:

a- editor window of the functions belonging to the variable weight 1; b- window of the editor of the functions belonging to the variable weight 2; c- the function editor window of the function membership variable; d- output Rule Editor window; e - output rule viewer window; f- solution Viewer window

When solving the problem (10), the number of "overclocking" solutions n was chosen equal to six: A_1, A_2, \dots, A_6 . Moreover, the extreme values A_1, A_6 were chosen at the boundaries of the area of change in the weighting coefficients α_1 and α_2 , and the average values A_2, A_3, \dots, A_5 were randomly generated. The results of solving the multi-criteria optimization problem are shown in Table 3 and illustrated in Fig. 3.

In Table 3, the «overclocking» iterations are highlighted in gray. The optimal value $\alpha_1 = 0.059$ and, respectively $\alpha_2 = 0.941$. At the same time, the values of the partial optimality criteria were: $f_1 = 38.560$ *centner/hectare* and $f_2 = 79023.222$ *thousand sum/hectare*. The optimal values of the variable parameters obtained during optimization

are distributed as follows: $x_1=6.0000$ thousand $m^3/hectare$; $x^3=0.2000$ Ton/hectare; $x^4=0.1500$ Ton/hectare.

The maximum time for one iteration when solving the problem (10) was 4.6 s. The total time for solving the problem (9) was 30 seconds.

Table 3. Optimization of problem solution results.

Iteration No.	α_1	α_2	F	ζ
1	0	1	1.99661	VW
2	0.31	0.69	1.99888	W
3	0.861	0.139	2.00295	B
4	0.599	0.401	2.00101	A
5	0.202	0.798	1.99809	W
6	1	0	2.00699	VB
7	0.08	0.92	1.99719	VW
8	0.04	0.96	1.99689	VW
9	0.059	0.941	1.99703	VW

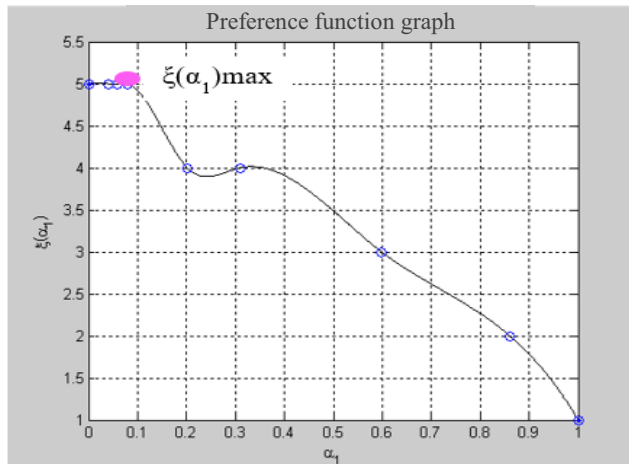


Fig. 3. Graph of the face preference function, the decision maker

4 Conclusion

1. The proposed method of solving the problem of optimizing the values of parameters of irrigation systems makes it possible to bypass the computational difficulties associated with the complexity and incorrectness of the problem.

2. The described algorithm of intellectual decision support makes it possible to solve the problem of multicriteria optimization of irrigation system parameters in conditions of uncertainty of priorities according to particular criteria of optimality.

3. The software implementation of the developed methodology and algorithm for optimizing the parameters of irrigation systems is very effective. It can be widely used in practice to solve the problems of multicriteria optimization of various irrigation systems in agriculture.

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