

# Calculation of compelled fluctuations of panel buildings

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**Abstract.** The article discusses the dynamic calculation of the elements of the box structure of buildings under the action of seismic effects, taking into account the spatial work of the box elements, given by the displacement of their lower part according to a sinusoidal law. The equations of motion for each of the plate and beam elements of the box structure of the building are constructed based on the Kirchhoff-Love theory. Expressions for the forces, moments, and stresses of the plate elements, which balance the box elements' displacement, the boundary conditions, and the conditions of full contact through displacements and force factors in the contact zones of the plate and beam elements, are compiled. The general solution to the problem is constructed by decomposing the movement of box elements according to their shapes using the finite difference method. The calculation results are shown in graphs for the height of the box of bending moments, plate elements working in bending, and shear stresses of plate elements.

## 1 Introduction

This article poses the problem of forced vibrations of a spatial box-shaped element of a building, consisting of rectangular panels and beam elements between them, as shown in Figure 1.

The work [1] is devoted to the method of static accounting of higher vibration modes in the problems of the dynamics of building structures under the action of an external harmonic load. With the help of the calculation software package, the displacements of nodes and internal forces in the elements of the structures under consideration are determined.

In [2], a multi-storey reinforced concrete frame-braced frame with prestressed girders is considered, subject to emergency impact, in the form of the sudden removal of a column of the outermost row on the first floor of the building. Using the finite element method, a nonlinear quasi-static analysis of deformation and failure was carried out under the structure in the form of a two-storey two-span frame separated from the building frame by the decomposition method.

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In a scientific study [3], oriented particle board (OSB) is considered an element of a pitched roof structure with a soft roof under the action of a vertical load from snow. The reasons for uneven deflections in OSB joints in the structures of inclined roofs and vertical walls are revealed, and recommendations for eliminating these reasons are justified.

The article [4] considers the impact of earthquakes of various intensities and frequency character on the seismic resistance of a wooden building. A computational and theoretical assessment of the frame building was carried out on simple and complex models under different intensities and frequency compositions. It has been established that the frequency composition of the seismic effect significantly affects the seismic resistance of frame buildings.

The article [5] proposes methods for solving dynamic problems of the dynamics of soils and earth structures and underground structures interacting with the soil.

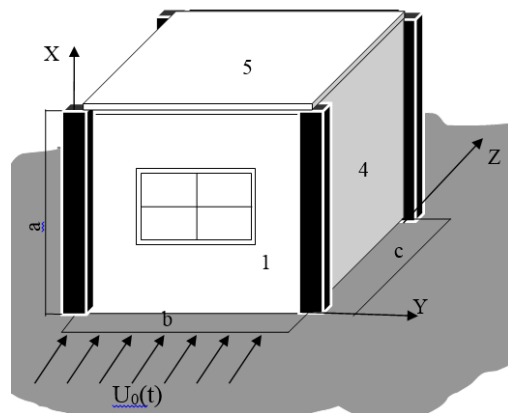
The authors of this article have performed several works using the theory of thin plates [6-8] and the bimoment theory of thick plates [9-14]. Scientific articles [6-8] are devoted to improving the box-shaped model of the building structure, considering the contact conditions between the elements of the panel and beams. The equations of motion of the box elements and the graphs of the displacements of plates and beams are constructed. The problems of forced vibrations of a building of the spatial box type are considered, consisting of rectangular panels and interacting bundles under dynamic action, given by the displacement of the base according to the sinusoidal law. In solving the problem, the method of finite differences was used.

In [15-16], the dynamic characteristics and oscillations of various axisymmetric and flat structures are considered, considering various geometries, spatial factors, and inelastic properties of materials. The problem's solution is carried out by the finite element method and expansion of the solution in natural vibration modes. Various mechanical effects associated with the structure's geometry and the material's inelastic properties are revealed.

The article deals with the problem of forced oscillations of a box-shaped structure of panel buildings (Figure 1), the bases of which oscillate according to a given law:

$$U_0 = A_0 \sin \varpi_0 t, \quad (1)$$

where  $A_0$  and  $\omega_0$  are the amplitude and frequency of forced oscillations. The work aims to determine the laws of change in displacements in the panels of the box of a building with a resonant mode.



**Fig. 1.** Box structure of the building.

A theoretical calculation of the box-shaped structure of large-panel buildings for dynamic effects was carried out under the assumption that the external load-bearing plate elements of the building 1 and 3, located perpendicular to the direction of seismic effects, work only for transverse dynamic bending in the OXY plane. Side plate elements 2 and 4, located in the direction of external influence, are subjected to tension-compression and shear in their plane OXZ

Let us introduce the following notation for displacement due to the deformation of plate and beam elements.

Plate deflections 1 and 3 working in bending are designated  $W(x, y, t)$ .

The displacements of plate elements working in shear are denoted  $u(x, z, t); v(x, z, t)$ . The plate elements of the box are connected by beam elements. Based on this condition, the beam elements are subjected to bending and torsion. Deflections and twist angles of beams will be denoted by  $W^{(i)}(x, t)$  and  $\alpha^{(i)}(x, t)$ , where:  $i$  is I, II, III, IV (number of beams).

It is assumed that the overlap (plate element 5) is also deformable. The law of movement of its points is determined following the forms of deformations of the upper edges of the vertical contacted plate elements, the functions of displacements of the points of overlap are designated  $u_n(z, y, t); u_n(z, y, t)$ .

The following designations for the plate elements of the building box are introduced in the article:  $E_b, \rho_b, b, h_b$ , and  $\nu_b$  are modulus of elasticity, density, width, thickness, and Poisson's ratio of bending plate elements,  $E_c, \rho_c, c, h_c$ , and  $\nu_c$  are modulus of elasticity, density, width, thickness and Poisson's ratio of shear plate elements.

$E_n, \rho_n, h_n$ , and  $\nu_n$  are the modulus of elasticity, density, thickness, and Poisson's ratio of the floor,  $G_n$  is the shear modulus of the floor  $G_n = \frac{E_n}{2(1+\nu_n)}$ .

The height of the box, i.e., the dimensions of all plate and beam elements in the vertical direction, are the same  $-a$

All beam elements are assumed to have a square section with dimensions  $h_b, h_b$  from the same material. The elastic and shear moduli of beams are denoted by  $E$  and  $G$ , Poisson's ratios  $\nu$  and density  $\rho$ , moments of inertia of the beam section in bending and torsion  $J$  and  $I_{kp}$ .

The forces and moments arising in the elements of the box and their connection zones are introduced. The expressions for the bending and torque moments of the bending plate elements  $M_{xx}, M_{yy}$ , and  $M_{xy}$  will be introduced by the formulas

$$\begin{aligned} M_{xx} &= -D \left( \frac{\partial^2 W}{\partial x^2} + \nu_b \frac{\partial^2 W}{\partial y^2} \right), \quad M_{yy} = -D \left( \nu_b \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} \right), \\ M_{xy} &= -D(1-\nu_b) \frac{\partial^2 W}{\partial x \partial y}, \end{aligned} \quad (2)$$

where:  $D = \frac{E_b h_b^3}{12(1-\nu_b^2)}$  is cylindrical stiffness in transverse bending,

The expressions for the longitudinal and tangential force of the plate elements working in shear can be represented as:

$$P_{zz} = B \left( \frac{\partial u}{\partial z} + \nu_c \frac{\partial v}{\partial x} \right), \quad P_{xx} = B \left( \nu_c \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right), \quad P_{xz} = \frac{B(1-\nu_c)}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \right). \quad (3)$$

where:  $B = \frac{E_c h_c}{1 - \nu_c^2}$  is cylindrical stiffness of plate elements in tension and compression.

Bending and torsional moments of beams are:

$$M^{(i)} = -EJ \frac{\partial^2 W^{(i)}}{\partial x^2}, \quad M_{kr}^{(i)} = EI_{kr} \frac{\partial \alpha^{(i)}}{\partial x}, \quad (4)$$

where:  $EI_{kp}$  is the torsional stiffness of the beam,  $EJ$  is the bending stiffness of the beam.

The expressions for the shearing force of the bending plates in the areas of their connections with the beams will be written as:

$$\begin{aligned} R_x &= Q_x + \frac{\partial M_{xy}}{\partial x}, \quad R_y = Q_y + \frac{\partial M_{xy}}{\partial y}, \\ Q_x &= -D \frac{\partial}{\partial x} (\Delta W), \\ Q_y &= -D \frac{\partial}{\partial y} (\Delta W). \end{aligned} \quad (5)$$

The longitudinal and tangential forces of the plate elements in the contact zones, working in shear, have the expressions:

$$P_z^c = B \left( \frac{\partial u}{\partial z} + \nu_c \frac{\partial v}{\partial x} \right)_{z=c_i}, \quad P_{zx}^c = \frac{B(1 - \nu_c)}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \right)_{z=c_i}. \quad (6)$$

where  $b_i$  and  $c_i$  are the corresponding coordinates of post-and-beam elements

The displacement field of post-and-beam elements during bending is determined based on the Bernoulli hypothesis in the form

$$v = -z \frac{\partial W}{\partial x}.$$

The torsion of the post-beam element is caused by the bending moments of the edges of the plate elements 1 or 3.

## 2 Method for solving the problem

An analytic-numerical method is proposed for solving the problem of vibrations of the building box, taking into account spatial deformations with full contact conditions in the zone of connections of the plate and beam elements of the building box.

Based on representation (1), we rewrite the kinematic laws of displacement of points of plate elements. The general kinematic law of box motion is presented as the sum of the base displacement function  $U_0(t)$  and the relative displacements of plates and beams

$$\begin{aligned} u_3 &= U_0(t) + W(x, y, t), \quad u_1 = U_0(t) + u(x, z, t), \quad u_2 = v(x, z, t), \\ u_3^{(i)} &= U_0(t) + W^{(i)}(x, y, t). \end{aligned} \quad (7)$$

We write the displacements of plate and beam elements in the form:

$$W = W(x, y) \sin(\omega_0 t),$$

$$u = u(x, z) \sin(\omega_0 t), v = v(x, z) \sin(\omega_0 t),$$

$$W^{(i)} = W^{(i)}(x, y) \sin(\omega_0 t), \alpha^{(i)} = \alpha^{(i)}(x, y) \sin(\omega_0 t). \quad (8)$$

Let us consider the theoretical calculation of the building box under dynamic action, considering the spatial work of the transverse and longitudinal plate elements.

Let us compose the equations of motion for each plate and beam element of the box structure of the building [6-8]. Lamellar elements are considered to be thin elastic plates obeying the Kirchhoff-Law hypothesis. Each beam is subject to bending and torsion. When constructing the equation of motion and the boundary conditions of plates and beams, we use the expressions for the moments of forces and displacement (2)-(6).

The system of equations for bending, torsional moments, and shear forces is presented in the form

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0, \quad (9.a)$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = 0,$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \rho h_b \ddot{W} + \rho h_b \ddot{U}_0. \quad (9.b)$$

The system of equations of motion of plate elements working in shear is taken as:

$$\frac{\partial P_{zz}}{\partial z} + \frac{\partial P_{xz}}{\partial x} = \rho h_c \ddot{u} + \rho h_c \ddot{U}_0, \quad (10)$$

$$\frac{\partial P_{zx}}{\partial z} + \frac{\partial P_{xx}}{\partial x} = \rho h_c \ddot{v}.$$

The system of equations for bending and torsional vibrations of beams has the form:

$$\frac{\partial M^{(i)}}{\partial x} - Q^{(i)} + \frac{h_b}{2} P_{zx}^c = 0, \quad (11.a)$$

$$\frac{\partial Q^{(i)}}{\partial x} + \rho F \ddot{W}^{(i)} = R_y - P_z^c - \rho F \ddot{U}_0, \quad (11.b)$$

$$\frac{\partial M_{kp}^{(i)}}{\partial x} = \rho I_{kp} \ddot{\alpha}^{(i)} + M_{yy}^b + \frac{h_c}{2} R_y. \quad (11.c)$$

where:  $GI_{kp}$  is the torsional rigidity of the beam.

Write the boundary conditions on the base of the building box as for a rigid pinch. The lower part of the building moves with the base, and there is no rotation.

$$u_1 = u_3 = u_3^{(i)} = U_0(t), u_2 = 0, \frac{\partial W}{\partial x} = 0, \frac{\partial W^{(i)}}{\partial x} = 0, \alpha^{(i)} = 0. \quad (12)$$

Boundary conditions (12) at  $x=0$ , taking into account (7), will be rewritten as:

$$W = 0, \frac{\partial W}{\partial x} = 0, u = 0, v = 0, W^{(i)} = 0, \frac{\partial W^{(i)}}{\partial x} = 0, \alpha^{(i)} = 0. \quad (13)$$

As boundary conditions at the upper ends of the elements of the building box at  $x=a$ , we accept the following contact conditions between these elements and the ceiling.

In the connection zone of the plate elements, full contact conditions are set, ensuring the equality of displacements and stresses. The contact conditions in the zone of joints of beam and plate elements working in shear will be written in the form

$$u(x, z, t)_{z=c_i} = W^{(i)}(x, t), v(x, z, t)_{z=c_i} = \pm \frac{h_b}{2} \frac{\partial W^{(i)}(x, t)}{\partial x},$$

$$W(x, y, t)_{y=b_i} = W^{(i)}(x, t), \left( \frac{\partial W(x, y, t)}{\partial y} \right)_{y=b_i} = -\alpha^{(i)}. \quad (14)$$

Let us denote the displacements of the upper points of beam and plate elements working bending and shearing

$$W_a(y, t) = W(a, y, t), u_a(y, t) = u(a, z, t), v_a(z, t) = v(a, z, t). \quad (15)$$

Based on the notation (15), the distribution law for the displacement of overlap points will be given by the expressions

$$u_n(z, y, t) = W_a(y, t) + u_a(z, t) - W^{(i)}(a, t)$$

$$v_n(z, y, t) = v_a(z, t). \quad (16)$$

Contact conditions at the joints of the floor and plate elements working in bending have the form

$$-R_x + \eta_0 \rho_n h_b h_n \ddot{W}_a = h_b h_n \frac{\partial \tau_{zy}^n}{\partial y} - \eta_0 \rho_n h_b h_n \ddot{U}_0, M_{xx}^a = 0. \quad (17)$$

where:  $\tau_{zy}^n = G_n \left( \frac{\partial W_a}{\partial y} \right)$  is shear stress of the overlap at its edge  $z = c$ ,  $\eta_0 = \frac{2(bh_b + ch_c)}{bc}$ .

The contact conditions at the junctions of the floor and post-and-beam elements will be written in the form

$$M^{(i)} = 0, -Q^{(i)} + \eta_0 \rho_n h_b h_n \ddot{W}^{(i)} = -\eta_0 \rho_n h_b h_n \ddot{U}_0, \quad (18. a)$$

$$M_{kr}^{(i)} = 0.$$

Contact conditions in the zones of butt joints of the floor and plate elements working in shear, relative to the contact tangential and normal stresses, will be written in the form

$$\begin{aligned}
 -ch_c \tau_{zx}^c + \eta_0 m_{nc} \ddot{u}_a &= ch_c h_n \frac{\partial \sigma_{zz}^n}{\partial z} - \eta_0 m_{nc} \ddot{U}_0, \\
 -ch_c \sigma_{xx}^c + \eta_0 m_{nc} \ddot{v}_a &= ch_c h_n \frac{\partial \sigma_{xz}^n}{\partial z}
 \end{aligned}
 \tag{18. b}$$

where:  $\sigma_{zz}^n = E_n \left[ \frac{\partial u_a}{\partial z} \right]$ ,  $\sigma_{zx}^n = G_n \left[ \frac{\partial v_a}{\partial z} \right]$  are normal and tangential stresses of the overlap on its edge,  $\sigma_{xx}^c = E_c \left[ \frac{\partial v}{\partial x} + \nu_c \frac{\partial u}{\partial z} \right]_{x=a}$ ,  $\tau_{zx}^c = G_c \left[ \frac{\partial v}{\partial z} + \frac{\partial u}{\partial x} \right]_{x=a}$  are normal and tangential stresses of plate elements working in shear, on its upper edge  $x = a$ .

The calculations were carried out for three values of the dimensionless frequency of the forced oscillation:  $\beta_1 = 16$ .  $\beta_2 = 17$ .  $\beta_3 = 18$  are with amplitude  $A_0 = 2$  cm.

Note that the above frequency values approach the value of the first natural frequency, and the movement of the building goes into a resonant mode. That is, the values of the frequencies of forced oscillations vary near the values of natural frequencies.

The following parameters are set as initial data:

the ratio of the heights of the panels to the width of the bent panel  $\frac{a}{b} = \frac{3}{4}$ . and the

height to the width of the shear panel  $\frac{a}{c} = \frac{3}{5}$ . The ratio of the thickness and width of the

bent panel  $\frac{H}{b} = \frac{0.28}{4}$ . and the thickness of the bent panel to the thickness of the shear panel

$\frac{H}{h} = \frac{0.28}{0.12}$ . The ratio of the moduli of elasticity of the bent panel and the shear panel  $\frac{E_1}{E_2} = \frac{3}{8}$ .

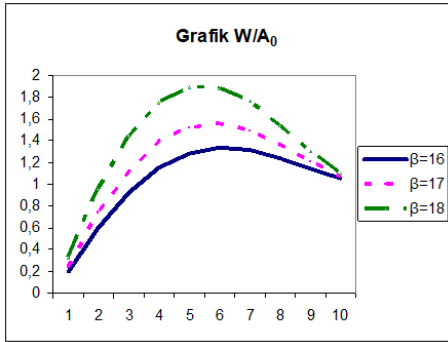
Poisson's ratio of the panel material  $\nu = 0.3$ . The material of beam elements and bending panels is the same. The transverse dimensions of the beam elements are equal to the thickness of the bent panel, respectively.

Figures 2 – 5 show the calculations of deflection and stress in panels and butt joints of slabs and box beams. The maximum deflection value was found at the second upper half of the height of the bent panel (Figure. 2).

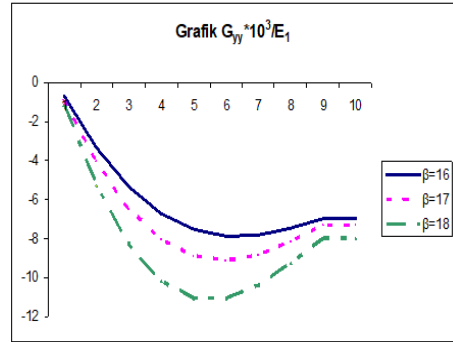
Figure 3 shows the plot of bending stress  $\sigma_{yy}$  along the height on the average vertical section of the panel. Figures 4 and 5 show changes in contact normal  $p_{zz}$  and shear stresses  $p_{xz}$ . The maximum value of the normal stress  $p_{zz}$  was found at the upper half of the height of the contact zone. It's compressive. The maximum contact shear stress  $p_{xz}$  was found in the lower part of the contact zone.

Calculations show that the maximum value of the contact shear stress is much higher than the value of the normal stress. At the junction between the inner slab and the bearing wall, rather large stress values were obtained.

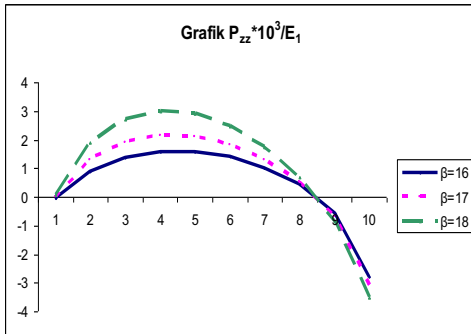
Laws of deflection change in height in the middle bending panel in resonant mode.



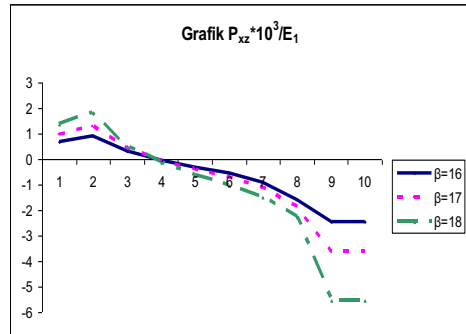
**Fig. 2.** Curves 1, 2, and 3 correspond to the values of the dimensionless deflection  $\bar{W} = \frac{W}{A_0}$  at the values of the dimensionless frequency of the external action -  $\beta$ .



**Fig. 3.** Curves 1, 2, and 3 correspond to normal stress values  $\bar{\sigma}_{yy} = \frac{\sigma_{yy}}{E_1} 10^3$  at the values of the dimensionless frequency of the external action -  $\beta$ .



**Fig. 4.** Curves 1, 2, and 3 correspond to the values of the normal contact stress  $\bar{P}_{zz} = \frac{P_{zz}}{E_1} 10^3$  at the values of the dimensionless frequency of the external action -  $\beta$ .



**Fig. 5.** Curves 1, 2, and 3 correspond to the values of the tangential contact stresses  $p_{xz} = \frac{\tau_{xz}}{E_1} 10^3$  at the values of the dimensionless frequency of the external action -  $\beta$ .

The law of change of the shear contact stress  $\tau_{xz}$  along the height of the butt joint of the beam element and the panel in the resonant mode.

For calculations, it was enough to accept  $n = m = 10$ .

### 3 Conclusions

1. The method of dynamic calculation of the box-shaped structure of buildings under dynamic influences has been developed. Equations of motion of points of plate and beam elements, boundary and contact conditions of the box of buildings of the problem of forced vibrations are constructed.

2. Using the method of finite differences, an analytical-numerical method for solving the problem of vibrations of the building box is proposed, taking into account spatial deformations with full contact conditions in the zone of connections of its plate and beam elements.



3. Within the framework of the finite-difference method, a method for dynamic calculation of the bending moments of plate elements of box-shaped structures of buildings has been developed.

4. From the graphs, it can be seen that the maximum values of the normal stress when taking into account the window openings are obtained by 25-30% more than the values of the normal stress obtained when solving the problem without taking into account the window openings.

5. In the zone of butt joints between the inner slab and the load-bearing wall, quite large stress values were obtained.

## References

1. Le T Q T, Lalin V.V., and Bratashov A.A., Static accounting of highest modes in problems of structural dynamics Magazine of Civil Engineering (St. Petersburg: Peter the Great St. Petersburg Polytechnic University) **88**, pp.3-13 (2019), DOI: 10.18720/mce.88.1
2. Kolchunov V.I., Fedorova N.V., Savin S.Yu., Kovalev V.V., Iliushchenko T.A. Failure simulation of a rc multi-storey building frame with prestressed girders // Magazine of Civil Engineering **8**(92). (2019)
3. Gavrilov, T.A., Kolesnikov, G.N. Oriented particle boards: Effect of the tangential load component. Magazine of Civil Engineering. 2017. DOI:10.18720/MCE.74.3.
4. Belash T.A., Ivanova Z. V. Timber frame buildings with efficient junction designs for earthquake-prone areas. Magazine of Civil Engineering. (2019) DOI:10.18720/MCE.92.7.
5. Bekmirzaev D.A., and Kishanov R.U. Assessment of the Effect of Inertia Forces in Problems of Underground Pipeline Seismodynamics. Int. J. Innov. Technol. Explor. Eng., **9**, pp. 500-503. (2020) DOI:10.35940/ijitee.C8526.019320
6. M. Usarov, G. Mamatisaev. Calculation on seismic resistance of box-shaped structures of large-panel buildings IOP Conf. Series: Materials Science and Engineering, **971** (2020) 032041. <https://doi.org/10.1088/1757-899X/971/3/032041>
7. M. Mirsaidov, M. Usarov, G. Mamatisaev. Calculation methods for plate and beam elements of box-type structure of building Building oscillations based on a plate model. E3S Web of Conferences **264**, 03030 (2021) <https://doi.org/10.1051/e3sconf/202126403030>
8. Usarov, M., Usarov, D., Mamatisaev, G. Calculation of a Spatial Model of a Box-Type Structure in the LIRA Design System Using the Finite Difference Method. Lecture Notes in Networks and Systems, **4036** (2022), [https://doi.org/10.1007/978-3-030-96383-5\\_141](https://doi.org/10.1007/978-3-030-96383-5_141)
9. Yarashov J., Usarov M., and Ayubov G. Study of longitudinal oscillations of a five-storey building on the basis of plate continuum model E3S Web of Conferences, **97** (2019) DOI:org/10.1051/e3sconf/20199704065
10. Toshmatov E., Usarov M., Ayubov G., and Usarov D. Dynamic methods of spatial calculation of structures based on a plate model E3S Web of Conferences **97**, (2019) DOI:org/10.1051/e3sconf/20199704072
11. Usarov D., Turajonov K., and Khamidov S. Simulation of free vibrations of a thick plate without simplifying hypotheses Journal of Physics: Conference Series **1425**, (2020) doi:org/10.1088/1742-6596/1425/1/012115

12. M. Mirsaidov and M. Usarov. Bimoment theory construction to assess the stress state of thick orthotropic plates *IOP Conf. Ser.: Earth Environ. Sci.* **614** (2020) <https://doi.org/10.1088/1755-1315/614/1/012090>
13. M. Usarov, A. Salokhiddinov, D. Usarov, I. Khazratkulov and N. Dremova. Tothetheoryof bending and oscillations of three-layered plates with a compressible filler. *IOP Conf. Series: Materials Science and Engineering* **869** (2020), doi:10.1088/1757-899X/869/5/052037
14. R. Abdikarimov, D. Usarov, S. Khamidov, O. Koraboshev, I. Nasirov and A.Nosirov. Free oscillations of three-layered plates. *IOP Conf. Series: Materials Science and Engineering* **883**, (2020), doi:10.1088/1757-899X/883/1/012058
15. M.Ishmatov, A.N.Mirsaidov. Nonlinear vibrations of an axisymmetric body acted upon by pulse loads, *Soviet Applied Mechanics* **27**(4), pp. 388-394, (1991). DOI: 10.1007/BF00896519
16. Mirsaidov M., Troyanovsky I. Forced axisymmetric oscillations of a viscoelastic cylindrical shell. *Polymer Mechanics.* **11**(6), pp. 953-955, (1975) DOI: 10.1007/BF00857626