# Damping of longitudinal vibrations of a cylindrical rod

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**Abstract.** Oscillation of an underground cylindrical rod located in an infinite medium and equipped with a dynamic vibration damper is considered. The vibrations of the structure occur from the action of harmonic forces located along the longitudinal axis of the bar. The possibility of decreasing the vibration amplitude using a two-mass absorber with a series connection of masses is considered. The parameters of the absorbers were optimized for different values of the environmental parameters. It was found that with a fairly high operating efficiency of the main damper for damping the first vibration mode, an additional damper can be used to reduce the amplitude of the second resonance. In this case, both dampers operate on separate vibration modes. Using a two-mass absorber can significantly reduce the formation of an underground structure. In this case, the expansion of the quenching zone was achieved in comparison with a single-mass quencher.

#### 1 Introduction

Underground structures are often affected, and there is a special interest in studying their behavior.

Depending on the type and direction of the impact of external loads on underground structures, they can be both in a state of extension and transverse risk. Transverse existence of a cylindrical structure in an elastic medium [1]. Variables changing in the value of the magnitude-frequency characteristic of buildings to alter environmental parameters when exposed to a low frequency. Longitudinal vibrations of a cylindrical rod in an elastic medium from the action of axial harmonic forces in a state of compression-tension were studied in [2].

Dynamic influences can create unfavorable vibrational states with large displacements. To create acceptable values of the vibration amplitudes of an underground structure, dynamic vibration dampers are used [3, 4]. In [3], the possibility of reducing the amplitude of oscillations of a hard disk in an elastic medium with the help of a multi-mass dynamic absorber was studied. The possibility of damping the oscillation amplitude of an underground cylindrical structure from the action of Rayleigh surface waves was considered in [4].

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Dynamic problems with physical and geometric nonlinearity are considered in [5, 6], and the dynamic stress state of structures in interaction with soil is studied in [7, 8]. Vibration problems of a finite element object are considered in [9, 10], and in [11], the bending of multilayer plates on an elastic half-space is studied, taking into account shear stresses.

Accounting for the inelastic properties of structural materials allows us to consider objects with internal energy damping and close to real mechanical properties [12-18].

The dynamic behavior of underground structures under the action of harmonic forces was studied in [19, 20].

Previous studies [3, 4] show the effectiveness of using absorbers to identify the likelihood of occurrence of underground phenomena. At the same time, a previously used single-mass absorber [4] is required to extinguish the accumulation of an underground structure, and to achieve a greater result, the use of a multi-mass absorber.

#### 2 Methods

In [2], the longitudinal oscillation of an underground structure from the action of harmonic forces applied along the longitudinal axis of the structure is considered. Dynamic equations are presented, and a formula for determining the displacement amplitude of a cylindrical underground structure is obtained. Using the obtained results, let us consider the vibrations of a cylindrical structure in an infinite medium equipped with a two-mass dynamic vibration damper. The underground structure is affected by harmonic forces applied along the longitudinal axis of the rod. Longitudinal oscillations of a cylindrical structure with dynamic vibration dampers are represented by the following dynamic equations.

$$E^*F\frac{d^2u_p}{dz^2} + 2\pi a\tau_{zz} - \rho F\frac{d^2u_p}{dt^2} - \frac{m_1}{L}\frac{d^2u_1}{dt^2} - \frac{m_2}{L}\frac{d^2u_2}{dt^2} = F(z,t),$$

$$m_1\frac{d^2u_1}{dt^2} + m_2\frac{d^2u_2}{dt^2} + \gamma_1\left(\frac{du_1}{dt} - \frac{du_p}{dt}\right) + k_1(u_1 - u_p) = 0,$$

$$m_2\frac{d^2u_2}{dt^2} + \gamma_2\left(\frac{du_2}{dt} - \frac{du_1}{dt}\right) + k_2(u_2 - u_1) = 0,$$
(1)

where,  $u_p(z,t)$  is longitudinal movement of the rod;  $E^*$  is complex Young's modulus  $E^* = E_0(1+i\delta)[15]$ ;  $\delta$  is parameter that takes into account attenuation in the bar; F, a,  $\rho$  are cross-sectional area and radius, bar specific gravity;  $\tau_{rz}$ —is shear stress of the medium on the line of contact with the rod;  $m_1, m_2$  are masses vibration dampers;  $k_1, k_2$  are respectively, the coefficients of stiffness and inelastic resistance of absorbers;  $u_1, u_2$  are respectively, the displacement of the masses of the absorbers.

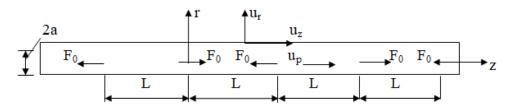


Fig. 1. Design scheme of an underground structure.

The first equation of system(0.1) (1) is transformed into the form

$$E^*F\frac{d^2u_p}{dz^2} + 2\pi\alpha\tau_{zz} - \rho F\frac{d^2u_p}{dt^2} (1 + \vartheta_{\varepsilon}M + \vartheta_n Q) = F(z,t), \quad (2)$$

where,

$$\mathbf{M} = \frac{u_1}{u_n}, \ \mathbf{Q} = \frac{u_2}{u_n}, \vartheta_{\varepsilon} = \frac{m_1}{L\rho F}, \vartheta_n = \frac{m_2}{L\rho F}. \tag{3}$$

The solution to equation (2) is represented in the following form [2]

$$u_p = \sum_{m=1}^{\infty} \frac{2aF_0}{FL} \frac{\cos\alpha_m z \cdot e^{-i\omega t}}{(2m-1)^2 (1+i\delta) - \alpha^2 \{1 - \frac{2\gamma_{pm}}{\Delta_m T [\gamma_{m}^2 + (2m-1)^2 (2\pi\theta)^2]}\}},$$
(4)

or

$$\frac{u_p FL}{2aF_0} = \sum_{m=1}^{\infty} \frac{2aF_0}{FL} \frac{\cos \alpha_m z \cdot e^{-i\omega t}}{(2m-1)^2 (1+i\delta) - \alpha^2 \left\{1 - \frac{2\gamma_{pm}}{\Lambda_m T [\gamma_{2m}^2 + (2m-1)^2 (2\pi m)^2]}\right\}},$$
 (5)

where,

$$T = \Omega \left[ 1 + \frac{\vartheta_{\varepsilon}(P - iN)}{1 + \vartheta^*} \right], P = \frac{l \cdot d + l \cdot p}{l^2 + t^2}, N = \frac{t \cdot d - l \cdot p}{l^2 + t^2}, l = \left[ (f_1^2 - q^2) - \vartheta^* q^2 U_1 \right],$$

$$t = (\mu_1 q + \vartheta^* q^2 V_1), d = \left[ f_1^2 + \vartheta^* (f_1^2 U_1 + \mu_1 q V_1) \right],$$

$$p = \left[ \mu_1 q + \vartheta^* (\mu_1 q U_1 - f_1^2 V_1) \right], U_1 = \frac{\left[ (f_2^2 - q^2) f_2^2 + (\mu_2 q)^2 \right]}{\left[ (f_2^2 - q^2)^2 + (\mu_2 q)^2 \right]},$$

$$V_1 = \frac{(\mu_2 q^3)}{\left[ (f_2^2 - q^2)^2 + (\mu_2 q)^2 \right]}, f_{1,2} = \frac{k_{1,2}}{m_{1,2} \alpha_0^2}, \mu_{1,2} = \frac{\gamma_{\varepsilon,n}}{m_{1,2} \alpha_0}.$$
(6)

Expression (4) is the equation of longitudinal vibrations of a cylindrical rod equipped with a multi-mass dynamic vibration damper. To reduce the amplitudes of the first and second resonances, it is necessary to optimize the parameters of the absorbers.

For a rod with an absorber, it is necessary to achieve  $u_p = 0$ . Consequently, the analytical condition for damping the resonance amplitude is the condition T = 0. From this, we obtain the equation

$$P^{2} + 2P \frac{(1+\vartheta^{*})}{\vartheta_{c}} + \frac{(1+\vartheta^{*})^{2}}{\vartheta_{c}^{2}} + N^{2} = 0.$$
 (7)

## 3 Results

The parameters entering into equation (5)  $f_1, f_2, \mu_1, \mu_2$  are optimizable, i.e., selecting a successful combination of these parameters (satisfying equation (5), we obtain the optimal damping of the resonant amplitude of the rod. The values  $\vartheta_2, \vartheta^*$  are fixed, optimization is performed for each value separately. The quality criterion is the maximum ordinate of the amplitude-frequency characteristic (AFC).

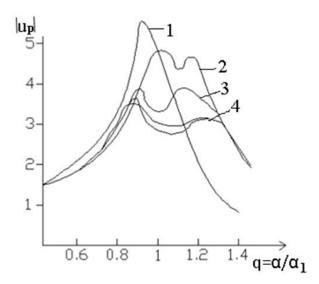
When considering different combinations of  $f_1$ ,  $f_2$ ,  $\mu_1$ ,  $\mu_2$  parameters, the optimal variant was chosen with the condition of minimizing the maximum ordinate of the frequency response in the resonant region.

When considering the damping of vibrations of the rod, the inelastic properties of the rod are taken into account ( $\delta = 0.2$ ).

## 4 Discussion

Let us first consider the operation of one (main) absorber, assuming. We will carry out the calculation for the case  $\vartheta^* = 0$ ,  $\Omega = 5$ , which expresses the relative density of the bar. We will also accept  $R_v = 0.5$ ,  $\eta = 0.10$ . Consider the mass of the main damper in three values -  $\vartheta_z = 0.01$ ; 0.05; 0.10.

As expected [3], with an increase in the absorber mass  $\theta_2$ , the quenching effect increases (Fig. 2). The optimal value of the partial frequency of the absorber, in this case, is 3-10% higher than the frequency of the rod. With increasing  $\theta_2$  the optimal value  $f_1$  approaches, and the optimal value of  $\alpha_1$  the attenuation coefficient  $\mu_1$  increases.



**Fig. 2.** AFC of the rod at  $\delta = 0.2$ ;  $\vartheta^* = 0$ ;  $1 - \vartheta_2 = 0$ ;  $2 - \vartheta_2 = 0.01$ ;  $3 - \vartheta_2 = 0.05$ ;  $4 - \vartheta_2 = 0.10$ 

The results of optimizing the damper parameters for the case  $\delta = 0.2$  are presented in table 1.

Table 1. Optimal parameters of a single-mass damper

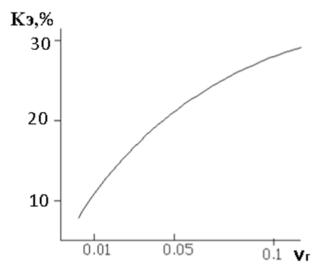
Absorber weight $\vartheta_{\mathrm{r}}$	Optimal parameters		Domning factor V	
	$f_1$	$\mu_1$	Damping factor $K_9$	
0.01	1.10	0.08	1.12	
0.05	1.06	0.14	1.23	
0.10	1.03	0.23	1.32	

The damping coefficient was determined by the formula

$$K_{9} = \frac{A_{o \max} + A_{e \max}}{A_{o \max}},$$

where,  $A_{o\,max}$  and  $A_{o\,max}$  are the maximum ordinates of the rod displacements, respectively, without and with an absorber.

Depending on  $\theta_{\varepsilon}$  the resulting damping effect reaches 12-35%. In fig. 3 shows the dependence  $K_{\vartheta}$  on the mass of the absorber  $\theta_{\varepsilon}$ .

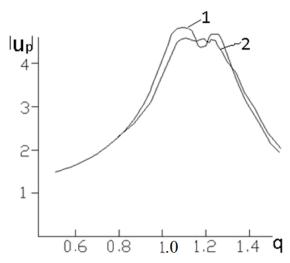


**Fig. 3.** Dependence  $K_{\vartheta}$  on  $\vartheta_{\varepsilon}$  at  $\delta = 0.2$ ;  $\vartheta^* = 0$ .

Suppose an additional (trimming) mass is connected to the main damper and the damping effect increases. The total mass of the two-mass damper is equal to the mass of the single-mass damper.

#### **5 Conclusions**

As is known from the research results [3], the introduced additional (tuning) mass expands the range of suppressed frequencies. The results of calculations carried out to study this effect led to the fact that the introduction of the second mass not only smoothes but also dampens the rod's resonant vibrations (Fig. 4).

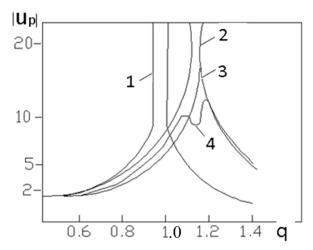


**Fig. 4.** AFC of the rod at  $\delta = 0.2$ ;  $\vartheta_z = 0.05$ ;  $1 - \vartheta^* = 0$  ( $K_3 = 23\%$ );  $2 - \vartheta^* = 0.01$  ( $K_3 = 30\%$ ).

As further studies of the system's frequency responsem are shown, the study of the vibrations of the rod without taking into account the inelastic properties of its material

 $(\delta = 0)$  establishes strong resonant vibrations with large, sometimes infinite amplitudes. Particularly strong vibrations occur when the impact frequency changes around the rod's first natural frequency. Since the first mode of oscillation is dangerous for such systems, it is necessary to extinguish the first resonance. Let us first use the main damper  $(\vartheta^* = 0)$ .

If the frequencies of the damper and the rod are equal  $(f_1 = 1)$  when optimizing  $\mu_1$ , we get a small effect (Fig. 5). It is interesting that for different  $\theta_2$  optimal values of the attenuation coefficient are the same -  $\mu_1 = 0.17$ .

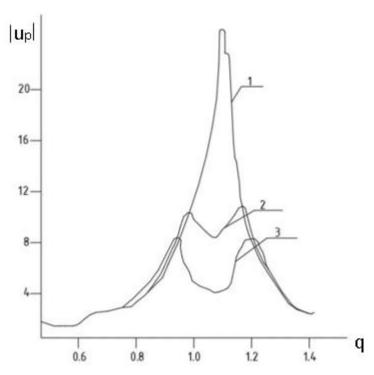


**Fig. 5.** AFC of the rod at  $\delta = 0$ ,  $\vartheta^* = 0$ ,  $f_1 = 1$ ,  $\mu_1 = 0.17$ ;  $1 - \vartheta_z = 0$ ;  $2 - \vartheta_z = 0.01$ ;  $3 - \vartheta_z = 0.05$ ;  $4 - \vartheta_z = 0.10$ .

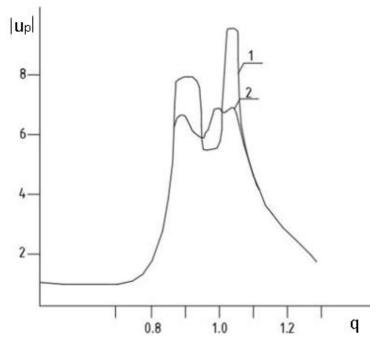
With optimization  $f_1$ ,  $\mu_1$ , it is possible to obtain a sufficiently high damping coefficient. In this case  $K_2$  is determined by the following dependence:

$$K_9 = \frac{A_{o max}}{A_{o max}}.$$

As shown in fig. 6, the frequency response graphs are relatively symmetrical. Depending on  $\theta_{\epsilon}$  the resonance amplitude can be reduced by 9-25 times, which is a very high result (Table 2). Although the main damper works quite efficiently, with the help of the second damper, it is possible to damp the oscillation amplitude even more, and most importantly, it is possible to smooth the frequency response in the resonant region, i.e., with a deviation of the frequency of exposure, the optimally tuned mass (tuning) does not allow the system to enter jump-like oscillations (fig. 7). Figure 7 can be seen that even a small adjustment mass ( $\theta^* = 0.01$ ) reduces the amplitude by 20% compared to the case  $\theta^* = 0$ .

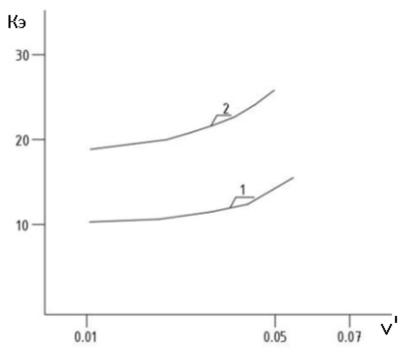


**Fig. 6.** AFC of the rod at  $\delta=0$ ;  $\vartheta^*=0$ ; 1-  $\vartheta_z=0.01$ ; 2-  $\vartheta_z=0.05$ ; 3-  $\vartheta_z=0.10$ .



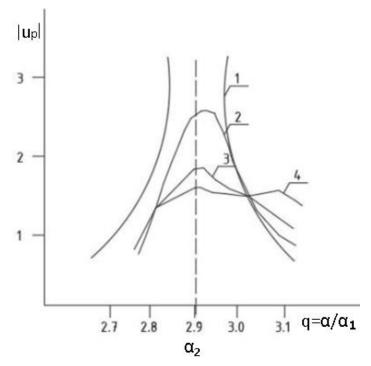
**Fig. 7.** AFC of the rod at  $\delta = 0$ ,  $\vartheta_c = 0.10$ ; 1- $\vartheta^* = 0$  ( $K_0 = 24.5\%$ ); 2- $\vartheta^* = 0.01$  ( $K_0 = 31.5\%$ ).

In fig. 8 shows the dependence  $K_9$  on the adjustment mass.



**Fig. 8.** Dependence  $K_2$  on  $\vartheta^*$  at  $\delta = 0$ ; 1-  $\vartheta_2 = 0.05$ ;  $2 - \vartheta_2 = 0.10$ .

When damping resonant vibrations of the rod, there may be another option. Since the main damper provides a fairly high damping effect for the first mode of vibration, the additional damper can be used to reduce the amplitude of the second resonance. In this case, both dampers operate on separate vibration modes. The results of such studies are shown in fig. 9. Although  $\vartheta^*$  fairly small value compared to  $\vartheta_{\varepsilon}$  but the achieved effect is large.



**Fig. 9.** AFC of the rod with the quenching of the second resonance,  $\vartheta_z = 0$ . 1- $\vartheta^* = 0$ ; 2- $\vartheta^* = 0.01$ ; 3- $\vartheta^* = 0.05$ ; 4- $\vartheta^* = 0.07$ .

Table 2 shows the data for the case of quenching two resonances when optimizing the parameters of the masses of the absorbers independently of each other.

1st resonance			2nd resonance				
$\vartheta_{\scriptscriptstyle \mathcal{Z}}$	$f_1$	$\mu_1$	$K_{2}$ ,%	$\vartheta^*$	$f_2$	$\mu_2$	$K_{\scriptscriptstyle 9},\%$
0.01	1.10	0.08	9.2	0.01	2.75	0.31	10.0
0.05	1.06	0.14	20.0	0.05	2.75	0.25	24.5
0.10	1.02	0.14	24.5	0.07	2.75	0.25	28.4

Table 2. Optimum damper parameters

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