

Strength of earth dams considering the elastic-plastic properties of soils

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Abstract. The article provides a detailed review of well-known publications devoted to studying the stress state and dynamic behavior of earth dams, taking into account the elastic and elastic-plastic properties of soil. To assess the stress-strain state of earth dams, considering the elastic-plastic properties of soil under the action of static loads, a mathematical model was developed using the principle of virtual displacements. A technique, algorithm, and computer programs were developed for estimating the stress state of dams using the finite element method and the method of variable elasticity parameters. A test problem was solved to assess the adequacy of the developed models and the reliability of the technique, algorithms, and computer programs. The stress-strain state of the Pskem earth dam, 195 m high, was studied using the developed models and calculation methods, taking into account the elastic-plastic properties of soil under the action of body forces and various levels of filling the reservoir with water. It was established that an account for the elastic-plastic properties of soil leads to a sharp change in the stress state of the dam, especially in the upper prism and in the core of the dam, and changes the intensity of normal stresses, which can lead to a violation of the integrity of the dam.

1 Introduction

The assessment of the stress-strain state and strength of earth dams are considered in the article, taking into account the elastic-plastic properties of soil under various impacts, i.e., under the action of mass forces and different levels of filling the reservoir with water.

Today, in the Republic of Uzbekistan, special requirements are stated for the safety of hydro-technical structures confirmed by the Law of the Republic of Uzbekistan On the safety of hydraulic structures, dated 08.20.1999, Resolution of the President of the Republic of Uzbekistan No. PP-4794 dated 07.30.2020 On measures on radical improvement of the system for ensuring seismic safety of the population and the territory of the Republic of Uzbekistan and the Law of the Republic of Uzbekistan On ensuring seismic safety of the population and the territory of the Republic of Uzbekistan dated September 13, 2021.

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Therefore, the development of design, construction, and operation of unique earth structures requires the creation of structures that work reliably under various kinds of loads. The existing elastic calculation, especially for high-rise structures, cannot cover the actual operation of the structure. Therefore, the development of new mathematical models and modern calculation methods that consider the nonlinear and plastic properties of soils, real geometry, non-homogeneity, and design features of the structure is relevant. Classical studies, which highlight the main theories and methods for assessing the strength parameters of earth structures, are given in [1-12].

Along with these publications, we should mention the following studies:

- in [13], natural oscillations of discrete systems were given, the difference between elastic-plastic oscillations and elastic ones was shown; it was shown that under elastic-plastic oscillations, the movement occurs according to an aperiodic law;
- in [14], numerical studies and evaluations of the seismic behavior of earth dams were conducted using the finite-difference method, considering the ideal plastic properties of soil and damping. The nonlinear dynamic behavior of the dam was investigated;
- the nonlinear seismic behavior of earth dams was considered in [15], using the finite element method and Geo-Studio software. In the numerical study, a nonlinear finite element analysis was used, taking into account the linear and elastic-plastic components of the model to describe the soil properties;
- in [16], the mitigation of the dam damage using a damping protective layer (a layer of river sand) between the foundation and the base was studied in the model of an earth dam under strong earthquakes;
- in [17], a unified analytical solution for the analysis of elastic-plastic stresses around a cylindrical cavity under biaxial stresses was presented. It was shown that the biaxial state of the initial stresses has a significant effect on the stress distribution around the internal cavity. The solutions obtained under loading and unloading were verified by comparison with the results of numerical simulation and other analytical solutions;
- in [18], the corresponding computational model and method were selected using the ABAQUS software package, and dynamic characteristics of a complex structure under strong dynamic impacts were analyzed. It was shown that the method of elastic-plastic analysis is more rational and reliable in assessing the behavior of structures and in checking calculations under strong dynamic impacts, and the calculated results are more accurate;
- elastic-plastic analysis of high-rise buildings under strong ground motions was performed in [19], using the ABAQUS software package. Anti-seismic characteristics were assessed, the structural scheme was optimized, and weak parts of the structure were strengthened;
- in [20], a nonlinear model was considered, taking into account the interaction of the structure with the soil base. Nonlinear effects were calculated based on an elastic-plastic model of the soil material adjacent to the structure's base. For the artificial boundary, non-reflective boundary conditions were used. A numerical method for solving the nonlinear equation of motion was developed;
- in [21], the method of gradual reduction of shear strength parameters was used to analyze the static stability of the slopes of earth dams based on numerical simulation. Strains in the body of the dam and base after the end of construction and the corresponding safety factor were modeled;
- the finite element method was used in [22] for the numerical calculation of earth dams; it considers the moisture content and plastic properties of soils and the nonlinear strain of the structure. Calculations were performed for three earth dams using the model proposed in the study;
- in [23], an improved elastic-plastic soil model was used to simulate the subsidence of high dams (182 m high) built from hard soils. The results showed a good agreement

between the calculated subsidence and the monitoring data. The stress-strain dependences were shown, and the results for volumetric strain were in better agreement compared to the results obtained using other models;

- in [24], the stability of slopes of earth dams was studied using an elastic-plastic model;
- the stability and displacement of an earth dam during construction were studied in [25] by mathematical modeling and experimental studies. The simulation results were compared with experimental data;
- in [26], the strain in the bodies was considered taking into account the elastic-plastic properties of the material and finite deformations. As an example, the tension of a rod of a circular cross-section was considered under elastic-plastic deformation;
- in [27], a technique for solving problems for a soil mass was presented, taking into account the elastic-plastic deformation of soil. As an example, the subsidence of an earth embankment was studied considering the elastic-plastic properties of soil;
- in [28], the pattern of strains and displacements of the body of the structure was studied by computer modeling;
- in [29], a two-dimensional problem of assessing the stress-strain state of a slope in an elastic-plastic formulation was solved using a numerical method;
- the strain state of water-saturated clay soils under triaxial cyclic loading was considered in [30], considering elastic-plastic deformation. It was shown that the calculated diagrams agree with the experimental strain diagrams of real soils.
- Along with this, the stress-strain state and dynamic behavior of various structures were studied in [31-43], taking into account the nonlinear, inelastic properties of the material and the features of their deformation.
- The above review of known publications shows that studying the stress-strain state of earth structures, considering the elastic-plastic properties of soil, is an urgent task.

2 Methods

2.1 Mathematical model of the problem

A non-homogeneous deformable system under the action of various static loads is considered (Fig. 1). It occupies volume $V=V_1+V_2+V_3$. Some elements of system V have elastic-plastic properties, and the other elements are elastic. It is necessary to determine the stress-strain state (SSS) of a non-homogeneous system (Fig. 1) under the action of body forces \vec{f} and the hydrostatic pressure of water \vec{p} . The problem is considered for the plane-strain state of the system (Fig. 1).

When setting the problem, it is assumed that the surface of the dam base Σ_0 is rigidly fixed, the areas of the crest and downstream slope Σ_2, Σ_3 are stress-free on the surface, and the hydrostatic water pressure \vec{p} acts on S_p , i.e., on the part of the surface Σ_1 .

It is necessary to determine the displacement and stress fields arising in the body of the dam (Fig.1) under the action of body forces \vec{f} and the hydrostatic pressure of water \vec{p} at various levels of reservoir fillings with water.

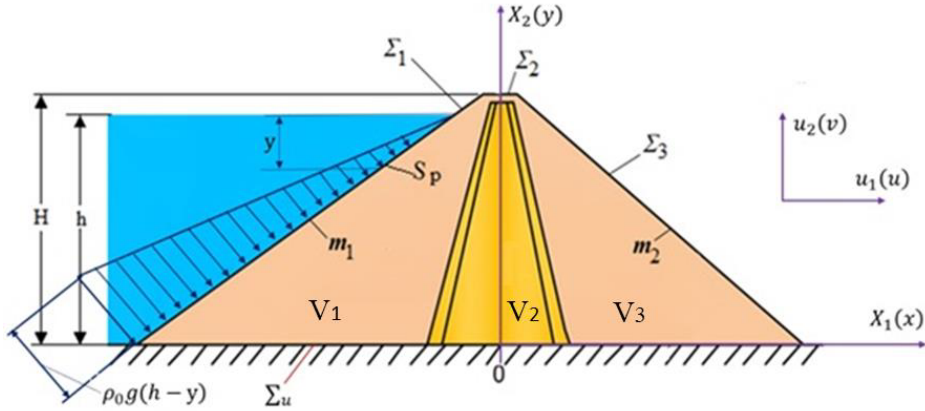


Fig. 1. Design scheme of a non-homogeneous system

To simulate the process of deformation in the body of the dam (Fig. 1), the principle of virtual displacements is used:

$$-\int_{V_1} \sigma_{ij} \delta \varepsilon_{ij} dS - \int_{V_2} \sigma_{ij} \delta \varepsilon_{ij} dS - \int_{V_3} \sigma_{ij} \delta \varepsilon_{ij} dS + \int_V \vec{f} \delta \vec{u} dS + \int_{S_p} \vec{p} \delta \vec{u} dS = 0. \quad (1)$$

Along with (1), the following values are used:

- kinematic boundary conditions

$$\vec{x} \in \sum_o : \vec{u} = 0 \quad (2)$$

- to describe the physical properties of the material in each area of the dam body (V_1 , V_2 , V_3), the generalized Hooke's law [44] is used:

$$\sigma_{ij} = \tilde{\lambda}_n \varepsilon_{kk} \delta_{ij} + 2\tilde{\mu}_n \varepsilon_{ij} \quad (3)$$

- to describe the relationship between the components of the strain tensor and displacement vector, the linear Cauchy relations are used [44]:

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad i, j = 1, 2 \quad (4)$$

The hydrostatic pressure of water on the upstream face of the dam is determined by the following formula [45]:

$$\vec{p} = \rho_o g (h - y) \quad (5)$$

Here \vec{u} are the components of the displacement vector, ε_{ij} , σ_{ij} are the components of the strain and stress tensors; \vec{p} is the hydrostatic water pressure; \vec{f} is the vector of

body forces; δ_{ij} is the Kronecker symbol; ρ_o is the density of water; $(h-y)$ is the depth of a point on the upstream face of the dam; $\vec{u} = \{u_1, u_2\} = \{u, v\}$ are the components of displacement vectors, $\vec{x} = \{x_1, x_2\} = \{x, y\}$ are the components of the coordinate system.

In the case of considering the law of elastic-plastic deformation of the materials of the dam, values $\tilde{\lambda}_n$ and $\tilde{\mu}_n$ are variables determined from the experiment for each section of diagram $\sigma_i = f(\varepsilon_i)$, and in the case of elastic deformations, they are the Lamé constants (index n shows the correspondence of the characteristic of the material to the part of the body - V_1, V_2, V_3);

When considering the elastic-plastic properties of the material, if at certain points of the dam body the stress intensity σ_i exceeds the yield strength σ_r (σ_r is determined from experiments for specific materials), then it is assumed that plastic deformations begin to develop in them due to a change in the shape of the body.

Now the problem under consideration can be formulated as follows: it is necessary to find the components of displacements $\vec{u}(\vec{x})$, strains $\varepsilon_{ij}(\vec{x})$, and stresses $\sigma_{ij}(\vec{x})$ under the action of body forces (\vec{f}) and hydrostatic water pressure \vec{p} , satisfying equations (1), (3), (4) and conditions (2) at all points of the dam body (Fig. 1) for any virtual displacement $\delta\vec{u}$.

2.2 Model implementation method

When implementing this approach, the virtual work of elastic forces must be rewritten in terms of the spherical and ninefold parts of stresses and strains [44]. Then the integrand in (1) can be represented as:

$$\sigma_{ij}\delta\varepsilon_{ij} = \sigma_i\delta\varepsilon_i + \frac{9}{2}K_n\delta\varepsilon_0^2 \quad (6)$$

Here $K_n = \frac{E_n}{3(1-2\nu_n)}$ is the volume modulus of elasticity, E_n is the modulus of elasticity, μ_n is the shear modulus of elasticity, ν_n is Poisson's ratio.

The first term in (6) is the virtual work produced by changing the shape, and the second term is due to the volume change.

The intensity of normal stresses σ_i and strains ε_i are determined by the following formulas:

$$\sigma_i = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6\sigma_{12}^2} \quad (7)$$

$$\varepsilon_i = \frac{\sqrt{2}}{2(1+\nu_n)} \sqrt{(\varepsilon_{11} - \varepsilon_{22})^2 + \varepsilon_{11}^2 + \varepsilon_{22}^2 + \frac{3}{2}\varepsilon_{12}^2}$$

In the case of plastic strains increment, the ninefold and spherical parts of the strains of corresponding stress components have the following form

$$e_{ij} = \frac{3}{2} \frac{\varepsilon_i}{\sigma_i} S_{ij} \quad (8)$$

$$\theta = 3\varepsilon_0 = \frac{\sigma_0}{K_n}$$

The relationship between the components of the strain tensor and the stress tensors can be written as [44, 48]:

$$\begin{aligned} \varepsilon_{11} &= \frac{1}{E_n^*} [\sigma_{11} - \nu_n^* \sigma_{22}] \\ \varepsilon_{22} &= \frac{1}{E_n^*} [\sigma_{22} - \nu_n^* \sigma_{11}] \\ \varepsilon_{12} &= \frac{1}{\mu_n^*} \sigma_{12} \end{aligned} \quad (9)$$

Here, S_{ij} , e_{ij} are the ninefold and σ_0, ε_0 are the spherical parts of stress and strain tensors; σ_i, ε_i are the stress and strain intensities; E_n^*, ν_n^*, μ_n^* are the variable elasticity parameters [46-48], determined through elastic parameters E_n, ν_n, μ_n , and intensities of normal stresses σ_i and strains ε_i i.e.:

$$E_n^* = \frac{\frac{\sigma_i}{\varepsilon_i}}{1 + \frac{1 - 2\nu_n}{3E_n} \frac{\sigma_i}{\varepsilon_i}}; \quad \mu_n^* = \frac{\sigma_i}{3 \cdot \varepsilon_i}; \quad \nu_n^* = \frac{\frac{1}{2} - \frac{1 - 2\nu_n}{3E_n} \frac{\sigma_i}{\varepsilon_i}}{1 + \frac{1 - 2\nu_n}{3E_n} \frac{\sigma_i}{\varepsilon_i}} \quad (10)$$

As seen from expression (10), the changing physical and mechanical parameters E_n^*, ν_n^*, μ_n^* at each structure point are determined based on the achieved strain state ε_i and the corresponding σ_i^* (according to the selected strain diagram (Fig. 2), i.e., $\sigma_i^* = \sigma_i^*(\varepsilon_i)$).

2.4 Test task

The reliability of the developed models, methods, algorithms, and computer programs was verified by studying the practical convergence when solving test problems.

The solution to a plane problem under the action of body forces for a homogeneous structure of an earth dam (Fig. 3) is considered, taking into account the elastic and elastic-plastic properties of soil. The problem is solved for the plane-strain state of the structure. The following values of geometric parameters of the dam and physical and mechanical properties of the soil material were taken: height - $H=86.5\text{m}$; the ratio of slope $m_1 = 1:2.5$, $m_2 = 1:2.2$; modulus of elasticity and specific gravity of soil $E=3.07 \cdot 10^4 \text{ MPa}$; $\gamma=1.98 \text{ t/m}^3$; Poisson's ratio - $\mu=0.36$; soil yield strength $\sigma_T = 5 \text{ MPa}$.

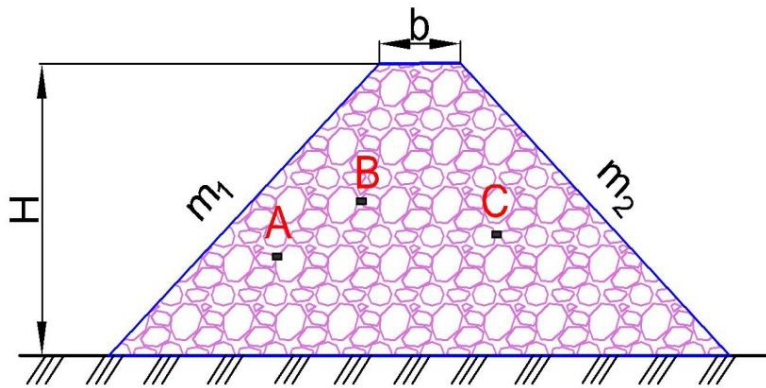


Fig. 3. Design scheme of a homogeneous earth dam

With the developed mathematical model and methods, the stress-strain state of the earth dam (Fig. 3) was determined under the action of body forces, taking into account the elastic and elastic-plastic properties of the dam material.

Vertical displacements u_2 , intensities of horizontal σ_{11} and normal stresses σ_i obtained at various partitions of the structure into finite elements for elastic and elastic-plastic cases are given in Table 1 for certain points (A, B, C) of the dam.

Table 1. The results obtained with various partitioning of the structure into finite elements

Points coordinate s (x,y)	Displacement and stress components	Number of finite elements		
		Elastic solution		
		N = 280	N = 336	N = 576
Point - A	u_2	$0.44336 \cdot 10^{-2}$	$0.44392 \cdot 10^{-2}$	$0.44441 \cdot 10^{-2}$
x=79.2m	σ_i	12.7045	12.7651	12.7908
y=28.8m	σ_{11}	-10.5147	-10.4173	-10.0337
Point - B	u_y	0.01634	0.01631	0.01606
x=208.8m	σ_i	70.2274	70.5124	70.7116
y=46.1m	σ_{11}	-20.7621	-20.7108	-20.6831
Point - C	u_2	0.01635	0.01639	0.01641
x=223.0m	σ_i	69.8439	70.0173	70.5069
y=40.1m	σ_{11}	-20.9777	-20.6428	-20.2835
Elastic-plastic solution				
Point - A	u_2	$0.5193 \cdot 10^{-2}$	$0.5204 \cdot 10^{-2}$	$0.5414 \cdot 10^{-2}$
x=79.2m	σ_i	16.206	16.673	16.888
y=28.8m	σ_{11}	-16.868	-16.274	-16.145
Point - B	u_2	0.03152	0.03231	0.03241
x=208.8m	σ_i	65.241	65.741	64.833
y=46.1m	σ_{11}	-17.188	-16.917	-16.781
Point - C	u_2	0.03154	0.03181	0.03192
x=223.0m	σ_i	64.281	64.301	64.254
y=40.1m	σ_{11}	-13.605	-12.912	-12.524

Checking the practical convergence of the results obtained (Table 1) of the problems under consideration for various numbers of finite elements shows good convergence of the results. In the elastic case, the convergence of the results occurs more rapidly than in the elastic-plastic case.

3 Results

The stress-strain state of the newly designed Pskem earth dam was studied; its height is $H = 195$ m, crest width - $b_{\text{crest}} = 12$ m, the ratio of slope $m_1 = 2.4$, $m_2 = 2.0$, the core width at the bottom - 130 m. Physical and mechanical properties of the core material are: specific gravity $\gamma = 1.7 \text{ t/m}^3$, deformation modulus $E_{\text{def}} = 30$ MPa, Poisson's ratio $\mu = 0.32$, internal friction angle $\phi = 24^\circ$, cohesion coefficient $c = 30$ kPa, yield strength $\sigma_T = 3$ MPa. Physical and mechanical properties of the material of retaining prisms are specific gravity $\gamma = 1.97 \text{ t/m}^3$, deformation modulus $E_{\text{def}} = 95$ MPa, Poisson's ratio $\mu = 0.27$, internal friction angle $\phi = 42^\circ$, cohesion coefficient $c = 70$ kPa, yield strength $\sigma_T = 5$ MPa. The data is taken from the project.

According to [50], $\alpha_{plast} = \frac{\alpha_{elast}}{3} = \frac{1}{3} \cdot \frac{3E}{2(1+\mu)} = \frac{1}{(1+\mu)}$. $\sigma_T = 3$ MPa (for loam material), $\sigma_T = 5$ MPa (for retaining prisms).

The calculation results of the Pskem earth dam, taking into account the non-homogeneous design features and elastic-plastic properties of soil under the action of body forces and hydrostatic pressure of water, are the definitions of the components of the displacement vector, the strain tensor, stress σ_{ij} , and stress intensity σ_i for all points of the considered area - of the cross-section of the dam.

To assess the effect of hydrostatic water pressure on the SSS of the dam, various levels of water filling in the reservoir were considered. The SSS obtained at each level of filling was compared with the results without considering the filling of the reservoir.

Figure 4 shows the isoline of distribution of equal values of the stress tensor components and the intensity of normal stresses obtained for the Pskem earth dam under the action of body forces, with account for the elastic and elastic-plastic properties of soil.

Solid (—) lines show elastic calculation, and dashed lines (-----) show the elastic-plastic calculation.

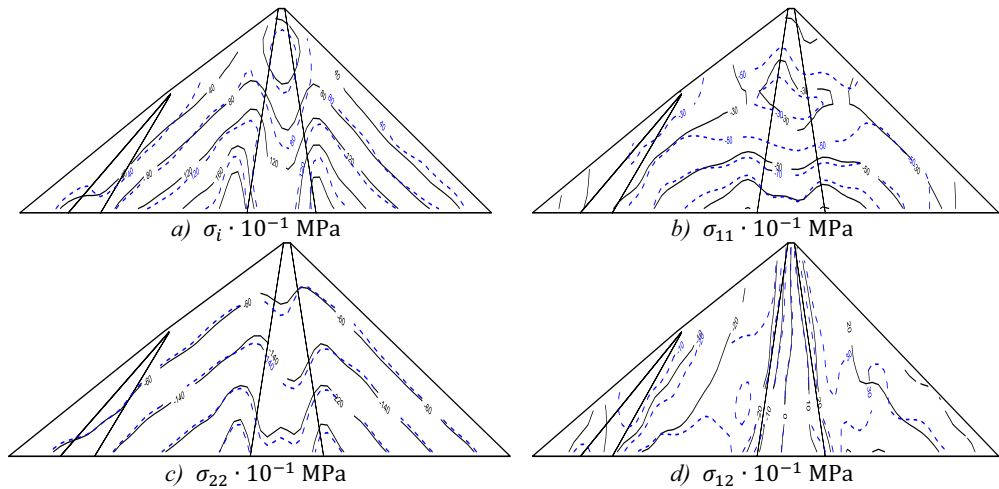


Fig. 4. Isolines of the distribution of equal values of the stress tensor components and the intensity of normal stresses in the Pskem earth dam under the action of body forces \vec{f} , taking into account elastic (—) and elastic-plastic (-----) properties of soil

The analysis of the results obtained (Fig. 4) shows the highest stress values σ_i , σ_{11} , σ_{22} , that occur in the middle of the lower part of the dam. Accounting for non-homogeneity, i.e., the presence of a core with other mechanical characteristics of soils leads to a decrease in stresses in the body of the core. Therefore, the distribution pattern of the isolines of these stresses in the dam's core goes down a little.

As for the isoline of distribution of tangential stresses σ_{12} , unlike other stresses, its most minimal value is observed approximately in the middle of the dam since the value of the ratio of both slopes is almost the same. The value of σ_{12} increases from the middle of the dam to the middle of the surcharge wall, then there is a slight decrease near the slope.

When considering the elastic-plastic properties of soil, the distribution of the intensity of normal stresses σ_i qualitatively differs little from the distribution in the elastic case, while quantitatively the value of this stress in some sections of the dam decreases to 17%.

When considering the plastic properties of soil, the greatest change occurs in horizontal stress σ_{11} in the middle part of the dam, it decreases to 30%.

Figure 5 shows the distribution of the isoline of equal values of the components of the stress tensor and the intensity of normal stresses obtained for the Pskem earth dam under the action of body forces and hydrostatic pressure of water when the reservoir is filled to a height $h = 190$ m, taking into account the elastic and elastic-plastic properties of soil.

An analysis of the results obtained for the dam under body forces and hydrostatic water pressure at various levels of reservoir filling showed a sharp change in the stress-strain state in the upper prism of the dam. If we compare the results given in Figs. 4 and 5, we can see a significant change in the stress-strain state of the dam. In this case, the stress is distributed asymmetrically in the upper retaining prism up to 2/3 of the dam's height. Then, considering the water pressure in the calculations, the intensity of normal stress σ_i in the upper retaining prism increases to 30%, and vertical stress σ_{22} increases to 50% for individual sections of the dam. The stresses σ_{11} , σ_{12} change substantially not only in the upper prism, but they change in the core of the dam; the symmetrical distribution of stresses σ_{12} throughout the body of the dam is completely broken.

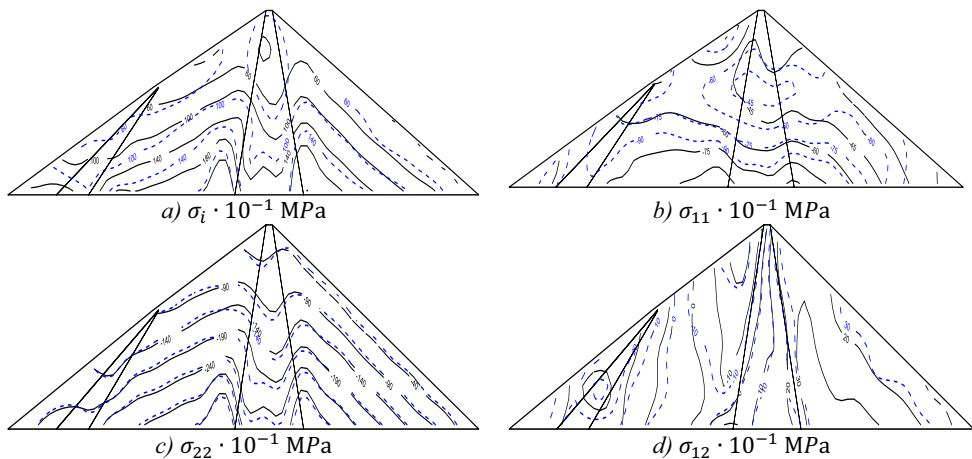


Fig. 5. Distributions of equal values of the stress tensor components (σ_{11} , σ_{12} , σ_{22}) and normal stress intensity σ_i in the Pskem earth dam under the action of body forces \vec{f} and water pressure \vec{p} when the reservoir is filled up to $h=190$ m, considering elastic (—) and elastic-plastic (---) properties of soil

An account for the elastic-plastic properties of soil leads to a decrease in stress from 10 to 50% in the upper prism and the core of the dam and from 5 to 30% in the lower prism and different parts of the dam.

4 Conclusions

1. A detailed review of the current state of the problem of assessing the stress-strain state of various dams is given, taking into account the nonlinear and elastic-plastic properties of the structure's material.
2. A mathematical model was developed for assessing the stress-strain of earth dams under various static effects using the principle of virtual displacements and small elastic-plastic strains occurring due to changes in the body's shape.

3. The methods and computer programs were developed for determining the stress state of earth dams using the finite element method and the method of variable elasticity parameters.

4. The effectiveness of the developed methods and algorithms for implementing the problem is shown by solving test problems.

5. The stress-strain state of the newly designed Pskem earth dam ($H=195$ m high) was studied under the action of body forces and hydrostatic pressures of water.

6. It was determined that:

- when considering the elastic-plastic properties of soil, the distribution of the intensity of normal stresses σ_i under the action of body forces qualitatively differs little from the distribution in the elastic case, while in different parts of the dam, it leads to a decrease in the stress state;

- under the action of body forces and hydrostatic water pressure on the dam at various levels of reservoir filling, an account for the elastic-plastic properties of soils leads to a sharp change in the stress state, especially in the upper prism and in the core of the dam. It changes the intensity of normal stresses up to 50%.

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