Stress-strain state and strength of earth dams under static loads

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Abstract. The article is devoted to studying the stress-strain state (SSS) and assessing the strength of the Gissarak, Sokh, and Pachkamar earth dams in a plane statement under the action of body forces and hydrostatic pressure. A mathematical model, algorithm, and computer program for studying the stress-strain state and assessing the strength of structures are developed in the article using the Lagrange variational equation. The finite element method was used in the problem solution. The adequacy of the model and the accuracy of the algorithm and program were verified by solving several test problems. It was revealed that the stress-strain state and strength of the studied earth dams significantly depend on the physical and mechanical properties of soils, the commensurability of geometric dimensions and slope coefficients of the dam retaining prisms, and on the level of water filling in reservoirs.

1 Introduction

In earth dams, a complex interaction occurs between the individual parts of the dam under the influence of its own weight and hydrostatic water pressure. In some cases, under the influence of these factors, tensile stresses appear in the body of the dam and its impervious devices, which can lead to crack formation and a violation of the strength characteristics of dams as a whole.

The task of studying the stress-strain state and assessing the strength of earth dams is a complex problem in the mechanics of a deformable rigid body; in solving this task, it is necessary to take into account the properties of materials, design features of structures, construction time, operation, variety of acting loads, etc. The solution to such a problem is difficult due to the lack of sufficiently substantiated data on the nonlinear and rheological properties of soils, the impossibility of simultaneous consideration of all possible factors in the numerical implementation of the solution, etc.

At the same time, the solution of particular problems with certain assumptions and prerequisites can be most fully and accurately obtained using numerical methods, for example, the finite element method (FEM) or the finite difference method (FDM) [1–5].

To date, there are a number of scientific articles devoted to studying the stress-strain state and assessing the strength of earth dams using various models of structures.

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Static and dynamic stress states of various soil dams are considered in [6–15]; these studies take into account the design features of structures, nonlinear and moisture properties of soils, the interaction of structures with the water environment of the reservoir, and other features of structures.

In [16], the data of time measurements of the dam's horizontal and vertical displacements is analyzed under its weight and water pressure in the reservoir of the Ikpoba River Dam (Benin). The calculation and correction of each measurement at different time points were performed separately. The results showed that nine points of the dam moved in a horizontal position, while at 10 points, the horizontal movements did not change. Vertical movement occurred in seven places, while in 13 points, the vertical movements did not change, both outside and along the dam's crest.

Reference [17] presents the results of a study in a plane formulation of assessing the stress-strain state of high earth dams depending on time (consolidation analysis). Some results of calculations to determine the effect of water pore pressure on the stress-strain state and subsidence of the dam were analyzed.

The study in [18] gives a systematic summary of the accumulated experience in constructing high earth rock fill dams. It discusses the main technical issues, including the control of deformations, seepage, slope stability, safety assessment, and other issues related to earth dams.

In [19], the stress state of earth dams under static and dynamic impacts was studied by the finite element method, taking into account the elastic-plastic strain of the dam soil, and the numerical results obtained were compared with the results of field measurements after the Wenchuan earthquake (China, 2008).

[20] analyzes in detail the use of non-traditional materials (soil and stone mixes) to ensure the slope stability of earth dams.

As the review shows, the studies of the stress-strain state and the assessment of the strength of earth dams, considering design features and actual operation of structures, were not sufficiently studied; therefore, research in this direction is of great scientific interest. The prediction of the behavior of earth dams should be based on the essential consideration of all factors affecting the SSS and strength under various types of loads.

Based on the above, this study is devoted to the development of the methods for calculating, in a plane statement, the SSS and determining the strength characteristics of the Gissarak, Sokh, and Pachkamar earth dams in Central Asia, taking into account the design features of the structure, the properties of materials and the level of reservoir filling. The finite element method was used as a computing tool.

2 Methods

Figure 1 shows a design diagram of an earth dam of complex geometry, occupying volume $V = V_1+V_2+V_3$ (V_1, V_3 are the volumes of the upper and lower prisms, V_2 is the volume of the core) rigidly fixed at the base Σ_u; the surfaces of the lower slope and crest are stress-free. An earth dam is under body forces $\vec{f}$, and hydrostatic water pressure $\vec{p}$ is applied to the surface Σ_1.

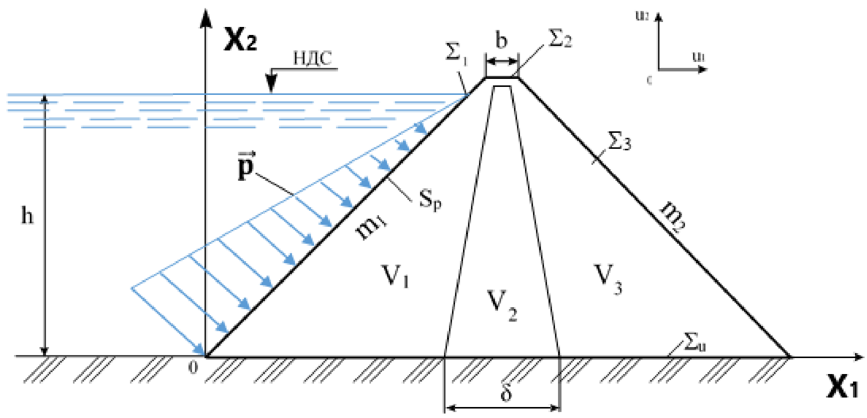


Fig. 1. Design diagram of an earth dam.

The principle of virtual displacements is used in the mathematical formulation of the problem; according to it, the sum of the work of all active forces acting on the system during virtual displacements is zero [5]:

$$\delta A=-\int_{V_1}\sigma_{ij}\cdot\delta\varepsilon_{ij}dV-\int_{V_2}\sigma_{ij}\cdot\delta\varepsilon_{ij}dV-\int_{V_3}\sigma_{ij}\cdot\delta\varepsilon_{ij}dV+ \\ +\int_{V}\vec{f}\cdot\delta\vec{u}\,dV+\int_{\Sigma_1}\vec{\mathrm{p}}\cdot\delta\vec{u}\,d\Sigma=0,\quad i,j=1,2 \tag{1}$$

The generalized Hooke's law [21] is used in the statement of the problem,

$$\left.\begin{aligned}\sigma_{11}&=\lambda\theta+2\mu\varepsilon_{11}\\ \sigma_{22}&=\lambda\theta+2\mu\varepsilon_{22}\\ \sigma_{12}&=\mu\varepsilon_{12}.\end{aligned}\right\} \tag{2}$$

$$\lambda=\frac{E\nu}{(1+\nu)(1-2\nu)},\quad \mu=\frac{E}{2(1+\nu)}$$

Cauchy ratio is

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), \tag{3}$$

and boundary conditions are

$$\vec{x}\in\sum{}_u:\quad \vec{u}=0 \tag{4}$$

The calculations consider different levels of the reservoir filling; the hydrostatic pressure of water on the upstream face of the dam is determined by the following formula

$$p = \rho_0 g(h - x_2) \qquad (5)$$

Here $\vec{u}$, ε_{ij}, σ_{ij} are the displacement vectors, components of strain and stress tensors, respectively; $\delta\vec{u}$, $\delta\varepsilon_{ij}$ are the isochronous variations of the components of displacement vector and strain tensors; ρ is the density of the material of the body; ρ_0 is the water density, (h –x_2) is the level of the reservoir filling, $\vec{f}$ is the vector of body forces; $\vec{p}$ is the hydrostatic water pressure acting on the surface Σ_1; λ and μ are the Lame constants; $\theta = \varepsilon_{kk}$ is the volumetric strain; $\{u_1,u_2\}=\{u,v\}$ are the components of the displacement vector of the body point; $\{x\}=\{x_1,x_2\}=\{x,y\}$ are the body point coordinates $i, j, k = 1,2$.

It is necessary to determine the functions of displacements $\vec{u}(\vec{x})$, strains $\varepsilon_{ij}(\vec{x})$, and stresses $\sigma_{ij}(\vec{x})$, arising in the body of the dam (Fig. 1) under the action of body $(\vec{f})$ and surface ($\vec{p}$) forces that satisfy equations (1)-(3) and boundary conditions (4) for arbitrary virtual displacements $\delta\vec{u}$.

To solve the variational problem (1) - (3) for a domain of a non-canonical complex form, the most appropriate is to use the finite element method (FEM), which allows taking into account both the geometry features and the properties of the structure material. Here, the area occupied by the body is divided into sub-domains with different physical and mechanical characteristics, then the sub-domains are automatically divided into first-order triangular finite elements with 6 degrees of freedom. As a result, a discrete model of the continuum is created.

To approximate the displacement field inside a triangular finite element, a linear function of the following form is used:

$$u_e = a_1 + a_2 x + a_3 y$$
$$\mathrm{v}_e = a_4 + a_5 x + a_6 y$$

where u_e, v_e are the horizontal and vertical displacements inside the *e*-th element, respectively; $a_1, a_2, ..., a_6$ are the unknown constants determined through nodal displacements and nodal coordinates of the *e*-th finite element.

The use of the finite element method procedure allows us to reduce the considered variational problem (1)-(4) to a system of high-order non-homogeneous algebraic equations:

$$[K]\{u\} = \{P\} \qquad (6)$$

Here $[K]$ is the stiffness matrix for the considered body (Fig. 1); $\{u\}$ are the sought-for components of the displacement vectors in the nodes of the finite element; $\{P\}$ are the components of external forces (mass and surface ones) acting on the nodes of the finite element.

In the study of specific variational problems, the partition of a given domain V into finite elements is performed, taking into account the design features and physical and mechanical properties of the material of different sections of the dam.

When solving the above tasks, the computer calculation program developed by the authors was used. The number of unknowns in equations (6) reached 5734.

3 Results and Discussion

The stress-strain state (SSS) and the strength of earth dams in a plane statement are considered in the article under the action of body forces and hydrostatic water pressure. When studying the effect of water on the SSS and the strength of dams, various levels of water filling (a gradual filling of the reservoir) were investigated.

Using the above mathematical model and method, the SSS and the strength of various dams under the action of water in the upstream face, and the own weight of the structure are studied, taking into account the actual physical and mechanical characteristics of soils and the design features of the dams under consideration.

The study was conducted for the following earth dams built in the regions of high seismicity of Central Asia, the main geometric and design features of which are as follows:

1) Gissarak dam (Fig. 2) with a height of H=138.5 m on the Aksu River in the Kashkadarya region of Uzbekistan, with slope coefficients m_{up}=2.2, m_{down}=1.9. Retaining prisms 1 are laid out of rock mass with the following physical and mechanical parameters - E=3600 MPa, soil specific gravity - γ=1.9 tf/m^3, Poisson's ratio - ν=0.3 and cohesion coefficient C= 2 kPa. Core 4 is laid from loam with the following physical and mechanical parameters - E=2400 MPa, soil specific gravity - γ=1.7 tf/m^3, Poisson's ratio - ν=0.35 and cohesion coefficient C= 20 kPa. The transition zone consists of sandy-gravelly soil. The crest of the dam is b=16m wide and L=660m long.

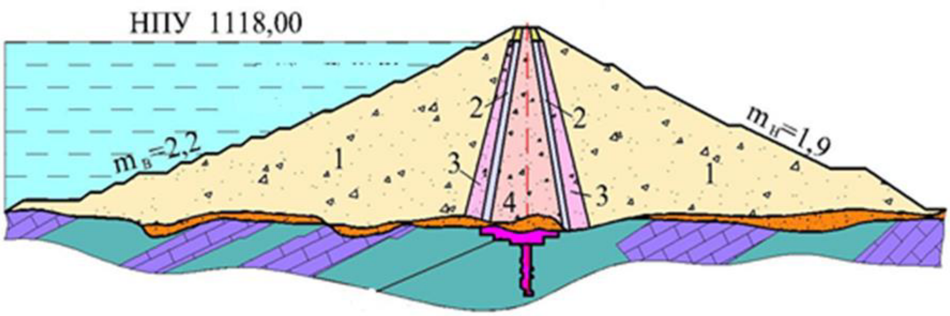


Fig. 2. Cross section of the Gissarak Dam**:** 1 is retaining prisms, 2 is 1st layer of transition zones, 3 is 2nd layer of transition zones, 4 is core.

2) Sokh dam, with a height of H=87.3 m, was built on the Sokh River in the Fergana region, with slope coefficients m_{up}=2.5, m_{down}=2.2. The retaining prisms are laid from pebbles with the following physical and mechanical parameters - E=3550 MPa, soil specific gravity - γ=2.1 tf/m^3, Poisson's ratio - ν=0.35 and cohesion coefficient C= 10.9 kPa. The core is laid from loam with the following physical and mechanical parameters - E=2400 MPa, soil specific gravity - γ=1.75 tf/m^3, Poisson's ratio - ν=0.35 and cohesion coefficient C= 30-50 kPa. The crest of the dam is b=10 m wide and L=487.3 long.

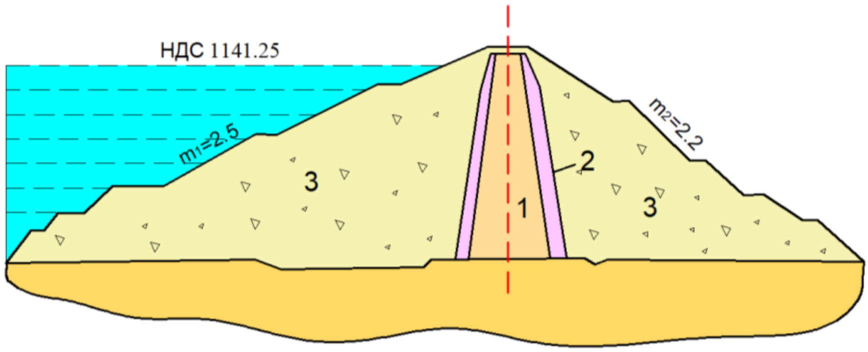


Fig. 3. Cross section of the Sokh Dam: 1 is core, 2 is two-layer transition zone, 3 is retaining prisms.

3) Pachkamar dam with height H=70 m was built in the Kashkadarya region, with slope coefficients m_{up}= m_{down}=2.25. The retaining prisms are laid from sandy-pebble stones with the following physical and mechanical parameters - E=3600 MPa, soil specific gravity - γ=2.25 tf/m^3, Poisson's ratio - ν=0.3 and cohesion coefficient C= 11 kPa. The core is laid from loam with the following physical and mechanical parameters - E=2400 MPa, soil specific gravity -γ=1.78 tf/m^3, Poisson's ratio - ν=0.35 and cohesion coefficient C= 30 kPa. The crest of the dam is b=8 m wide and L=589 m long.

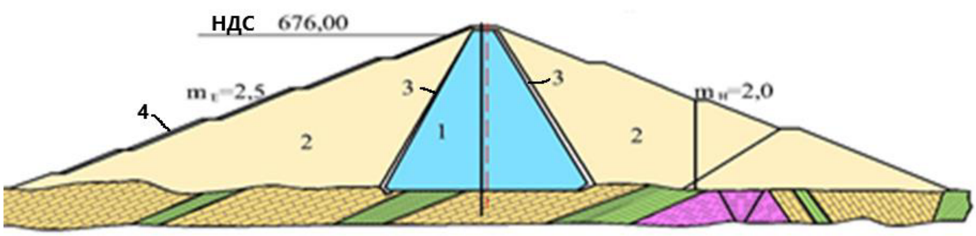


Fig. 4. Cross section of the Pachkamar dam: 1 is core, 2 is retaining prisms, 3 is transition zones, 4 is monolithic concrete pavement

The problem is solved in two stages: at the first stage of the calculation, the SSS of earth dams under the action of their own weight and hydrostatic water pressure is considered; and at the second stage, using the results for the stresses obtained at the first stage of the calculation, the strength of the considered dams is estimated using the Coulomb-Mohr strength theory.

The results of the calculation are the components of displacement u, v, strains ε_x, ε_y, γ_{xy} and stresses σ_x, σ_y, τ_{xy} vectors, as well as the safety factor *"K"* for all points of the structure.

At the first stage of the calculation, the SSS is determined for the central main cross-section of the dam in a plane statement. For the convenience of analyzing the results, the isolines of the displacement and stress components are constructed in the cross-section of the dam.

Figure 5 shows the fields of equal levels of horizontal (a, c) and vertical (b, d) movements of the Gissarak dam in a plane statement under its own weight and hydrostatic pressure of the reservoir water.

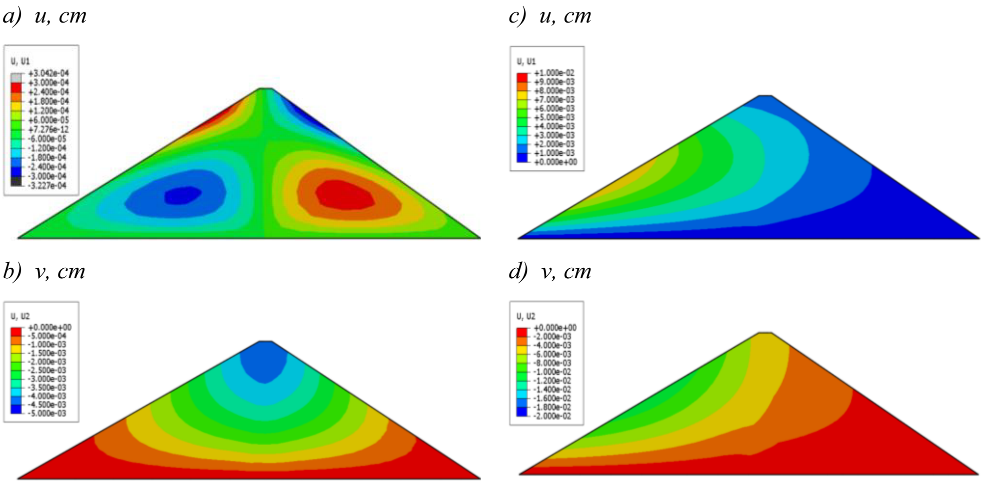
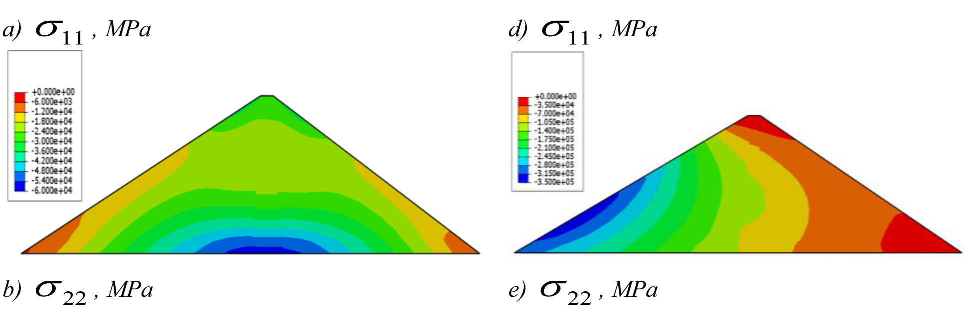


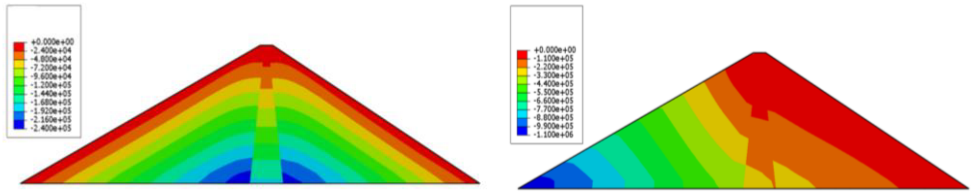
Fig. 5. Fields of equal levels of displacements (u and v) of the Gissarak dam at the empty reservoir (a, b) and full-filled reservoir (c, d).

The analysis of the results obtained (Figs. 5, a and b) shows that the displacements of points in the body of the dam are approximately symmetrical concerning the vertical axis of the dam. In the dam's core, the value of horizontal displacements is close to zero, and their value increases towards the centers of the upper and lower retaining prisms. The displacements of the point in the vertical direction prevail. This is explained by the fact that the calculations were made considering only the own weight of the dam. At the points located in the structure's upper levels, the displacement values are greater than at the points of the lower level. The greatest displacements are observed on the crest and in the zone of the dam core. Consideration of design features, namely the presence of a core laid of loam, significantly affects the value of displacements only in the core zone.

The values of the displacements of the Gissarak dam profile points (Figs. 5, c, and d) significantly depend on the level of water filling in the reservoir. With an increase in the water level, the displacement field of the dam profile also changes: when the reservoir is half-filled, only the displacement field of the upper supporting prism changes; with an increase in the filling level, the displacement field gradually changes in the core and then in the lower supporting prism. Similar patterns were observed in the study of the SSS of the Sokh and Pachkamar dams.

Figure 6 shows fields of equal horizontal σ11 (a), vertical σ22 (b), and tangential σ12 stresses of the Gissarak dam under its own weight and hydrostatic water pressure at an empty and completely filled reservoir.





c) σ_{12} *, MPa* *f)* σ_{12} *, MPa*

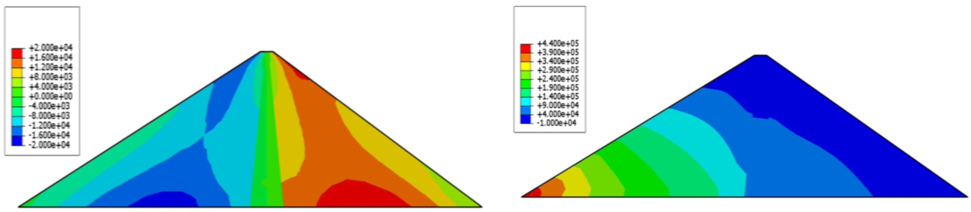


Fig. 6. Fields of equal levels of horizontal σ_{11} (a, d), vertical σ_{22}(b, e), and tangential σ_{12} (c, f) stresses of the Gissarak Dam at empty (a, b, c) and completely filled reservoir (d, e, f).

The obtained results show that the stresses are practically zero in the zones close to the dam contour, which is explained by the absence of load on the surface of the crest and slopes. The SSS is almost symmetrical about the vertical axis of the dam. The lines of zero level of tangential stresses σ_{12} pass along the central axis of symmetry of the dam. With distance from this axis, the values of σ_{12} increase, reaching a maximum in the lower part of the slopes. At the same time, the influence of the design features of the core leads to the appearance of an arch effect and a significant change in the SSS of the dam. These phenomena are explained by the fact that due to differences in the deformability of materials of the loamy core and retaining prisms caused by static or dynamic loads, an arch effect is observed in the cross-section at the contact of the core with transition zones and prisms. As a result, the solidity (the monolithic character) of the dam may be broken with the probable formation of through cross cracks inside the core and longitudinal cracks on the crest

A comparison of the results obtained shows that when the reservoir is filled, the effect of hydrostatic water pressure changes the pattern of distribution of stresses σ_{11}, σ_{22}, and σ_{12} in the body of the dam, and their symmetrical nature is completely lost. In this case, the value of stresses σ_{11} increases by 2-3 times, and vertical stresses σ_{22} increases up to 2 times in areas close to the slope in the upper prism.

In the second stage of the calculation, using the results of stresses obtained at the first stage, the strength of the considered dams in a plane formulation is estimated using the Coulomb-Mohr strength theory. According to this theory, under plane strain, the safety factor *"K"* at each point of the dam is determined by the following expression [21]

$$K = \frac{0.5(\sigma_1+\sigma_2)\sin\varphi + c\cdot\cos\varphi}{0.5(\sigma_1-\sigma_2)} \quad (7)$$

Here *K* is the safety factor at each point of the dam; φ is the angle of internal friction of soil; ***c*** is the cohesion coefficient; σ_1 and σ_2 are the values of the main stresses arising at the points of the dam.

Depending on the value of the determined safety factor *"K"*, the following conclusions are drawn in the study:

1) If $K > 1$, then these points of the earth dam are considered stable.

2) If $K = 1$, then these points of the earth dam are within the strength limit.

3) If $K < 1$, then the strength at these points of the dam is considered unsecured, and the dam's stability is lost in these areas.

To assess the strength of the Gissarak, Sokh, and Pachkamar earth dams under the influence of their own weight and hydrostatic water pressure with the completely filled reservoir, using expression (7), the values of the safety factor *"K"* at all points of the body of the dams were determined, and the strength of the dams under consideration was evaluated. The results obtained are presented in Fig. 7.

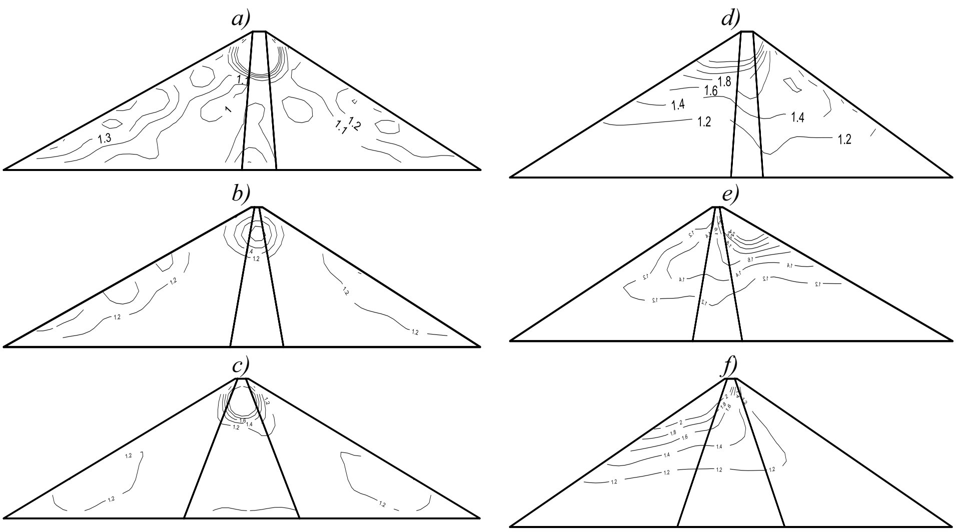


Fig. 7. Distribution lines of the safety factor *"K"* in the body of the Gissarak (a, d), Sokh (b, e), and Pachkamar (c, f) earth dams under their own weight (a, b, c) and under the influence of their own weight and completely-filled reservoir (d, e, f)

The value of the safety factor of these dams (under the action of static loads) in all sections is $K>1$, which shows the strength and stability of the dams as a whole. It was also determined that the hydrostatic water pressure in the reservoir greatly influences the pattern of distribution of the safety factor *"K"* in the body of the dam.

4 Conclusions

1. To study the stress-strain state and assess the strength of earth dams in a plane statement under the action of static loads, a mathematical model was developed based on the variational Lagrange equation. The set problems were reduced to a high-order non-homogeneous system of algebraic equations using the finite element method.

2. Using the developed calculation program, the SSS was investigated, and the strength of three different dams was assessed under the influence of their own weight and hydrostatic pressure of water using the Coulomb-Mohr strength theory.

3. It was determined that:

- displacements of points in the dam body under its own weight were approximately symmetrical concerning the vertical axis of the dam. At that, the movement of points in the vertical direction prevailed. At the points located in the structure's upper levels, the displacement values were greater than at the points of the lower level. The greatest displacements were observed on the crest and in the zone of the dam core;

- when studying the SSS of earth dams, it is necessary to take into account the design features, i.e., the strain properties of the core, since this fact significantly affects the strength of the stress-strain state of the structure;

- the level of water filling in the reservoir has a significant impact on the stress-strain state of the dam body; the maximum effect was observed when the reservoir was completely filled.

4. Determination of the strength of earth dams using the Coulomb-Mohr theory of strength shows that the values of the safety factor *"K"* for all points of the considered dams turned out to be $K > 1$. It follows that the strength of these earth dams is provided by body forces and hydrostatic pressure.

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