

Implementation of the Floyd-Warshall Algorithm and Bellman-Ford Algorithm to Determine the Shortest Path in the Distribution of LPG Gas

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Abstract. In this article, the method use to determine the shortest path in the distribution of LPG is the Floyd-Warshall algorithm and the Bellman-Ford algorithm. These two methods will be compared to find out methods that can provide the best result. The best result is the path that has the minimum distance of distribution of LPG. Both of these algorithms are focused on simulating calculations that begin by representing the map in the form of a graph. Finally, the results obtained from this study are that using the Floyd-Warshall algorithm produces a path with a total distance of 111.05 KM while the Bellman-Ford algorithm produces 332.3 KM.

1 Introduction

In the current era, the increasing needs of human life demand a higher level of production in every existing company [1]. Basic needs such as LPG gas which is the main thing in the household. The scarcity of LPG gas from the level of production to distribution is certainly one of the main problems in every LPG company. As in PT.TOYUNGO, many bases must be addressed so that it takes a longer time in the distribution process. Therefore, determining the path for distribution is certainly one of the most important factors to make product distribution efficient.

Mathematics is a tool that can be applied to solve problems and simplify a presentation. One of the fields of mathematics is graph theory. A graph is a set of vertex that are joined by a edge that has weights. The set of several vertices that meet several edges is also called a graph [2]. Graph theory is a topic that has useful models for its application so that it gets a lot of attention. Solving problems such as transportation, communication networks, operations research, computer science, etc [3]. The most specific problems that can be modeled using graphs include determining the path in the distribution of services/goods. Distribution for one location point to another location point by paying attention to cost efficiency or distance is carried out at any time, as a result, in the delivery of goods there are often delays to consumers [4]. In graph theory, there are several algorithms for solving the shortest path, as used in this study, namely the Floyd-Warshall algorithm and the Bellman-Ford algorithm. The Floyd-Warshall algorithm can guarantee success in determining the solution because

in its application it uses a dynamic program to determine the shortest path to find the optimum solution for all pairs of nodes. In representing a graph, the Floyd-Warshall algorithm uses a matrix of a graph. In a weighted graph, the Bellman-Ford algorithm can calculate the shortest distance (from a single source). That is, from one source, all the shortest distances originating from one vertex/node are calculated. The advantage of a single source is that this algorithm measures all the shortest distances that occur from a vertex.

Nowadays, some research to determining the shortest path using the Floyd-Warshall algorithm have been studied, such as determining the location of based tower stations, waste transportation routes, and distribution of goods [4–7]. In several other studies, the shortest path to determine cases such as distribution of goods, routes to gas stations and hospitals was studied using the bellman-ford algorithm [1, 8–10] and then, Other studies have also compared the two and implemented them to determine the optimal route, including in determining the shortest route to the museum in Jakarta [11] and to an android-based national university [12].

This study will determine the shortest path in the distribution using the Floyd-Warshall algorithm and Bellman ford algorithm. This research focused on the distribution of LPG Gas at PT. Toyungo in Gorontalo. PT Toyungo experienced problems with the long distribution time due to the large number of bases owned. Therefore, it is necessary to determine the optimal solution in the distribution process to be able to streamline distance and time. In determining the shortest path solution, Floyd-Warshall's algorithm guarantees success more because this algorithm applies to a dynamic program. This algorithm can compare every edge of all vertex that is passed to determine the path in the graph [3]. The Bellman-Ford

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Table 1. Labeling Vertex

V	Description	V	Description	V	Description
V ₁	PT. TOYUNGO	V ₁₈	Molosipat W	V ₃₅	Bugis
V ₂	Liluwo	V ₁₉	Limba U 2	V ₃₆	Talumolo
V ₃	Pulubala	V ₂₀	Limba U 1	V ₃₇	Leato Utara
V ₄	Paguyaman	V ₂₁	Limba B	V ₃₈	Leato Selatan
V ₅	Dulalowo Timur	V ₂₂	Biau	V ₃₉	Botu
V ₆	Wangkaditi Barat	V ₂₃	Biawao	V ₄₀	Tamalate
V ₇	Tomulabutao Selatan	V ₂₄	Donggala	V ₄₁	Moodu
V ₈	Tomulabutao	V ₂₅	Siendeng	V ₄₂	Dembe 2
V ₉	Huangobotu	V ₂₆	Tenda	V ₄₃	Dembe Jaya
V ₁₀	Tuladenggi	V ₂₇	Pohe	V ₄₄	Wangkaditi
V ₁₁	Libuo	V ₂₈	Tanjung Kramat	V ₄₅	Dulomo
V ₁₂	Buladu	V ₂₉	Ipilo	V ₄₆	Dulomo Selatan
V ₁₃	Dembe 1	V ₃₀	Heledulaa S.	V ₄₇	Tanggikiki
V ₁₄	Lekobalo	V ₃₁	Heledulaa	V ₄₈	Bulotadaa Timur
V ₁₅	Pilolodaa	V ₃₂	Wumialo	V ₄₉	Bulotadaa
V ₁₆	Buliide	V ₃₃	Dulalowo	V ₅₀	Molosipat U
V ₁₇	Tenilo	V ₃₄	Padebuolo	V ₅₁	Tapa

Algorithm can handle negative weights on the melting of the shortest distance in a graph [7]. So that these two methods will be implemented to determine the optimal path in the distribution of LPG gas at PT. TOYUNGO.

In this paper we investigate the shortest path of distribution of LPG gas from PT. TOYUNGO as an initial vertex to destinations vertex by using the algorithm Floyd Warshall and algorithm Bellman-Ford. As far as we know, this case is on PT. TOYUNGO has never been studied and investigated. This paper is organized as follows: In Section 2, we give the methods used in our research. In Section 3, all of our research results are presented and discussed. Finally, the conclusion of our research is given in Section 4.

2 Research Method

The model that will be studied is a model in the case analysis of LPG gas distribution, namely distribution carried out by PT. TOYUNGO to existing gas bases in the city of Gorontalo. The initial model built on this algorithm is a weighted connected graph, with weight in the form of mileage used to determine the shortest path.

The shortest path of this model is investigated and presented using several methods as follows.

- Representing the map in the form of a graph as well as labeling each vertex and edge.
- Identify the shortest path using both algorithms.
- Determine the most optimal solution in determining the path for distribution of LPG Gas

3 Result and Discussion

3.1 Data Representation in the Graphs

In this chapter, we begin by showing the range of locations that includes PT. Toyungo and LPG gas bases as distribution destinations. We connect each adjacent vertex then interpret it into a graph and we get Figure 1.

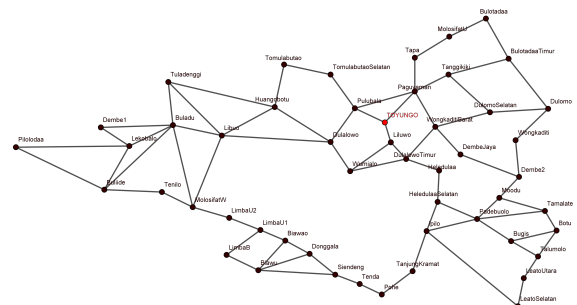


Figure 1. Representation of base vertex in graph

Where, PT. TOYUNGO as the main vertex and the total of all destination vertex is 51 vertices which represents the number of LPG Gas distribution bases. To make it easier, the next step is to label each vertex that exists and with the help of Google MAPS, the distance between the connected vertex is obtained. The labeling of vertex and distances is shown in Table 1 and Table 2, respectively.

After data interpretation is obtained, in the next subsection we start to identify the shortest path by using both algorithms.

3.2 The Implementation of Floyd-Warshall Algorithm

In this part, we investigate the shortest path of the distribution LPG gas using the Floyd warshall algorithm [13, 14].

Table 2. The distance between vertex

Point	Description	Distance(km)	Point	Description	Distance(km)	Point	Description	Distance(km)
e_1	V_1-V_2	0,85	e_{27}	$V_{12}-V_{13}$	5	e_{53}	$V_{29}-V_{38}$	6,7
e_2	V_1-V_3	0,55	e_{28}	$V_{12}-V_{14}$	3,3	e_{54}	$V_{30}-V_{31}$	1,2
e_3	V_1-V_4	0,45	e_{29}	$V_{12}-V_{16}$	2,3	e_{55}	$V_{30}-V_{34}$	1,7
e_4	V_2-V_5	0,45	e_{30}	$V_{12}-V_{18}$	2,3	e_{56}	$V_{32}-V_{33}$	1
e_5	V_2-V_{32}	1,9	e_{31}	$V_{13}-V_{14}$	5,3	e_{57}	$V_{34}-V_{35}$	1,5
e_6	V_3-V_4	1	e_{32}	$V_{14}-V_{15}$	1,3	e_{58}	$V_{34}-V_{40}$	2,2
e_7	V_3-V_7	1,4	e_{33}	$V_{14}-V_{16}$	3,1	e_{59}	$V_{34}-V_{41}$	2,2
e_8	V_3-V_{33}	1,1	e_{34}	$V_{15}-V_{16}$	4,1	e_{60}	$V_{35}-V_{36}$	1,5
e_9	V_4-V_6	3,1	e_{35}	$V_{16}-V_{17}$	2,7	e_{61}	$V_{35}-V_{39}$	2
e_{10}	V_4-V_{47}	1,3	e_{36}	$V_{17}-V_{18}$	2,1	e_{62}	$V_{36}-V_{37}$	6,4
e_{11}	V_4-V_{51}	0,75	e_{37}	$V_{18}-V_{19}$	2,1	e_{63}	$V_{36}-V_{39}$	2,8
e_{12}	V_5-V_6	2,5	e_{38}	$V_{19}-V_{20}$	1,2	e_{64}	$V_{37}-V_{38}$	4,3
e_{13}	V_5-V_{31}	2	e_{39}	$V_{20}-V_{21}$	1,3	e_{65}	$V_{39}-V_{40}$	2,1
e_{14}	V_5-V_{32}	1,6	e_{40}	$V_{20}-V_{23}$	1,1	e_{66}	$V_{40}-V_{41}$	1,8
e_{15}	V_6-V_{43}	1	e_{41}	$V_{21}-V_{22}$	1	e_{67}	$V_{41}-V_{42}$	1,2
e_{16}	V_6-V_{46}	2,5	e_{42}	$V_{22}-V_{23}$	0,75	e_{68}	$V_{42}-V_{43}$	2,2
e_{17}	V_7-V_8	1,1	e_{43}	$V_{22}-V_{24}$	1,7	e_{69}	$V_{42}-V_{44}$	1,5
e_{18}	V_8-V_9	2,2	e_{44}	$V_{22}-V_{25}$	1,8	e_{70}	$V_{44}-V_{45}$	3,2
e_{19}	V_9-V_{10}	1,7	e_{45}	$V_{23}-V_{24}$	2,1	e_{71}	$V_{45}-V_{46}$	1,2
e_{20}	V_9-V_{11}	1,6	e_{46}	$V_{24}-V_{25}$	1,7	e_{72}	$V_{45}-V_{48}$	2,5
e_{21}	V_9-V_{33}	2,2	e_{47}	$V_{25}-V_{26}$	1	e_{73}	$V_{46}-V_{47}$	3,7
e_{22}	$V_{10}-V_{11}$	2,9	e_{48}	$V_{26}-V_{27}$	3,4	e_{74}	$V_{47}-V_{48}$	1,2
e_{23}	$V_{10}-V_{12}$	1,5	e_{49}	$V_{27}-V_{28}$	1,9	e_{75}	$V_{48}-V_{49}$	1,4
e_{24}	$V_{11}-V_{12}$	2,1	e_{50}	$V_{28}-V_{29}$	6,3	e_{76}	$V_{49}-V_{50}$	1,1
e_{25}	$V_{11}-V_{18}$	2,2	e_{51}	$V_{29}-V_{30}$	0,8	e_{77}	$V_{50}-V_{51}$	1
e_{26}	$V_{11}-V_{33}$	1,9	e_{52}	$V_{29}-V_{34}$	1,4			

where, V represents the vertex, and E represents the distance between the connected vertex.

By using the data we have, we start the iteration by paying attention to the initial matrix $K = 0$ which is formed from the initial value and then interpreted into the form of the matrix.

The search for the shortest path using the Floyd-Warshall algorithm is carried out by an iterative process with $K = (1-51)$. By following a similar way with [13, 14] we obtained the final result in matrix $K = 51$ that can be seen in Figure 4 at Appendix 1.

From this result, By looking at the smallest weight between each pair of vertexs, the shortest path is obtained, namely $V_1 - V_4 - V_{51} - V_{50} - V_{49} - V_{48} - V_{47} - V_{46} - V_{45} - V_{44} - V_{42} - V_{43} - V_6 - V_5 - V_{31} - V_{30} - V_{34} - V_{41} - V_{40} - V_{39} - V_{35} - V_{36} - V_{37} - V_{38} - V_{29} - V_{28} - V_{27} - V_{26} - V_{25} - V_{24} - V_{23} - V_{22} - V_{21} - V_{20} - V_{19} - V_{18} - V_{17} - V_{16} - V_{15} - V_{14} - V_{13} - V_{12} - V_{10} - V_{11} - V_9 - V_8 - V_7 - V_{33} - V_{32} - V_2$. The shortest path that can be traversed is shown in Figure 2.

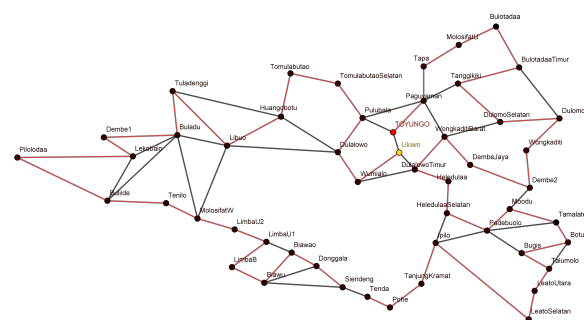


Figure 2. The Shortest Path

formed from the path with the smallest weight connecting vertex.

3.3 The Implementation of Bellman-Ford Algorithm

By using the Standard Bellman-Ford Algorithm [15], we start to calculate the data we have from the first iteration until the 10th iteration where all vertex has been passed and the calculation results are obtained in the Figure 3.

To check whether the obtained path is the optimum path, the authors compare the obtained path with other paths shown in Table 3.

Based on Table 3 it can be seen that the path that has been obtained previously is the path with the smallest weight when compared to other paths because the path is

Iterasi	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25		
0	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
1	0	0,85	0,55	0,45	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
2	0	0,85	0,55	0,45	1,3	3,55	1,95	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
3	0	0,85	0,55	0,45	1,3	3,55	1,95	3,05	3,85	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
4	0	0,85	0,55	0,45	1,3	3,55	1,95	3,05	3,85	5,55	5,65	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
5	0	0,85	0,55	0,45	1,3	3,55	1,95	3,05	3,85	5,55	5,65	10,65	8,95	∞	7,95	7,85	5,75	7,85	∞	∞	∞	∞	∞	∞	∞		
6	0	0,85	0,55	0,45	1,3	3,55	1,95	3,05	3,85	5,55	5,65	10,65	8,95	10,25	7,95	7,85	5,75	7,85	9,05	∞	∞	∞	∞	∞	∞		
7	0	0,85	0,55	0,45	1,3	3,55	1,95	3,05	3,85	5,55	5,65	10,65	8,95	10,25	7,95	7,85	5,75	7,85	9,05	10,35	∞	∞	10,15	∞	∞		
8	0	0,85	0,55	0,45	1,3	3,55	1,95	3,05	3,85	5,55	5,65	10,65	8,95	10,25	7,95	7,85	5,75	7,85	9,05	10,35	11,35	∞	10,15	12,25	∞		
9	0	0,85	0,55	0,45	1,3	3,55	1,95	3,05	3,85	5,55	5,65	10,65	8,95	10,25	7,95	7,85	5,75	7,85	9,05	10,35	11,35	∞	10,15	12,25	13,15		
10	0	0,85	0,55	0,45	1,3	3,55	1,95	3,05	3,85	5,55	5,65	10,65	8,95	10,25	7,95	7,85	5,75	7,85	9,05	10,35	11,35	∞	10,15	12,25	13,15		
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51		
∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
∞	∞	∞	∞	∞	∞	∞	2,75	1,65	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
∞	∞	∞	∞	∞	∞	∞	3,3	2,75	1,65	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
∞	∞	∞	∞	∞	4,5	3,3	2,75	1,65	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
∞	∞	∞	∞	5,3	4,5	3,3	2,75	1,65	6,2	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
∞	∞	∞	11,6	5,3	4,5	3,3	2,75	1,65	6,2	7,7	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞		
∞	∞	∞	13,5	11,6	5,3	4,5	3,3	2,75	1,65	6,2	7,7	9,2	16,3	12	9,7	8,4	7,95	6,75	4,55	8,65	5,45	5,45	1,75	2,95	3,3	2,2	1,2
16,9	13,5	11,6	5,3	4,5	3,3	2,75	1,65	6,2	7,7	9,2	16,3	12	9,7	8,4	7,95	6,75	4,55	8,65	5,45	5,45	1,75	2,95	3,3	2,2	1,2		
16,9	13,5	11,6	5,3	4,5	3,3	2,75	1,65	6,2	7,7	9,2	16,3	12	9,7	8,4	7,95	6,75	4,55	8,65	5,45	5,45	1,75	2,95	3,3	2,2	1,2		
14,15	13,5	11,6	5,3	4,5	3,3	2,75	1,65	6,2	7,7	9,2	16,3	12	9,7	8,4	7,95	6,75	4,55	8,65	5,45	5,45	1,75	2,95	3,3	2,2	1,2		

Figure 3. The Final Result of Bellman-Ford Algorithm

Table 3. The Alternatif Path

No.	The Path	Weight
1	$v_1 - v_2 - v_{32} - v_5 - v_{31} - v_{30} - v_{34} - v_{41} - v_{40} - v_{39} - v_{35} - v_{36} - v_{37} - v_{38} - v_{29} - v_{28} - v_{27} - v_{26} - v_{25} - v_{24} - v_{23} - v_{22} - v_{21} - v_{20} - v_{19} - v_{18} - v_{17} - v_{16} - v_{15} - v_{14} - v_{13} - v_{12} - v_{10} - v_{11} - v_{33} - v_9 - v_8 - v_7 - v_3 - v_4 - v_{51} - v_{50} - v_{13} - v_{12} - v_{10} - v_{11} - v_{33} - v_9 - v_8 - v_7 -$	111,95
2	$v_1 - v_2 - v_{32} - v_5 - v_6 - v_{43} - v_{42} - v_{44} - v_{45} - v_{46} - v_{47} - v_{48} - v_{49} - v_{50} - v_{51} - v_4 - v_3 - v_7 - v_8 - v_9 - v_{33} - v_{11} - v_{10} - v_{12} - v_{13} - v_{14} - v_{15} - v_{16} - v_{17} - v_{23} - v_{24} - v_{25} - v_{26} - v_{27} - v_{28} - v_{29} - v_{38} - v_{37} - v_{36} - v_{35} - v_{39} - v_{40} - v_{41} - v_{34} - v_{30} - v_{31}$	112,45

Table 4. The path of Bellman-Ford Algorithm

No.	The Path
1	$v_1 - v_2 - v_{32} - v_5 - v_{31} - v_{30} - v_{34} - v_{41} - v_{40} - v_{39} - v_{35} - v_{36} - v_{37} - v_{38} - v_{29} - v_{28} - v_{27} - v_{26} - v_{25} - v_{24} - v_{23} - v_{22} - v_{21} - v_{20} - v_{19} - v_{18} - v_{17} - v_{16} - v_{15} - v_{14} - v_{13} - v_{12} - v_{10} - v_{11} - v_{33} - v_9 - v_8 - v_7 - v_3 - v_4 - v_{51} - v_{50} - v_{49} - v_{48} - v_{47} - v_{46} - v_{45} - v_{44} - v_{42} - v_{43} - v_6$
2	$v_1 - v_2 - v_{32} - v_5 - v_6 - v_{43} - v_{42} - v_{44} - v_{45} - v_{46} - v_{47} - v_{48} - v_{49} - v_{50} - v_{51} - v_4 - v_3 - v_7 - v_8 - v_9 - v_{33} - v_{11} - v_{10} - v_{12} - v_{13} - v_{14} - v_{15} - v_{16} - v_{17} - v_{18} - v_{19} - v_{20} - v_{21} - v_{22} - v_{23} - v_{24} - v_{25} - v_{26} - v_{27} - v_{28} - v_{29} - v_{38} - v_{37} - v_{36} - v_{35} - v_{39} - v_{40} - v_{41} - v_{34} - v_{30} - v_{31}$
3	$v_1 - v_4 - v_{51} - v_{50} - v_{49} - v_{48} - v_{47} - v_{46} - v_{45} - v_{44} - v_{42} - v_{43} - v_6 - v_5 - v_{31} - v_{30} - v_{34} - v_{41} - v_{40} - v_{39} - v_{35} - v_{36} - v_{37} - v_{38} - v_{29} - v_{28} - v_{27} - v_{26} - v_{25} - v_{24} - v_{23} - v_{22} - v_{21} - v_{20} - v_{19} - v_{18} - v_{17} - v_{16} - v_{15} - v_{14} - v_{13} - v_{12} - v_{10} - v_{11} - v_9 - v_8 - v_7 - v_3 - v_{33} - v_{32} - v_2$
Total	332,3 Km

Based on Figure 3, we obtained several shortest-path options that can be taken in the distribution of LPG gas at PT. TOYUNGO in Table 4.

Therefore, from the bellman ford algorithm, we obtained three routes for the distribution of LPG Gas on PT. TOYUNGO.

4 Conclusion

The shortest path for distribution of the LPG Gas of PT. TOYUNGO by using the implementation of Floyd Warsall algorithm and Bellman-Ford Algorithm have been investigated. Based on the results and discussion, we know that the implementation of the two algorithms can be applied in determining the shortest route in the LPG Gas Distribution at PT TOYUNGO, however, it can be seen that there are differences in the iteration process of these two algorithms where in the iteration process the Bellman Ford Algorithm is completed faster because the iteration process stops when all vertexs have been passed. While

the floyd-warshall algorithm in the iteration process continues until $K = 51$. Due to differences in the iteration process, there are also differences in the results obtained where the Floyd-Warshall algorithm has the shortest route with a total weight of 111.05 Km while the Bellman-Ford algorithm's total weight is obtained 332.3 Km.

References

- Ester Tetri H Hutasoit, Jurnal Sistem Komputer dan Informatika (JSON) **1**, 20–25 (2019)

2. Demitra Januar Bawole, Hanna Prillysca Chernovit, INOBIS: Jurnal Inovasi Bisnis dan Manajemen Indonesia **3**, 41–51 (2019)
3. Barahama, Romario Marselino, Christie EJC Montalu, Rinancy Tumilaar, Jurnal Matematika dan Aplikasi **10**, 31–36 (2021)
4. Nawagusti, Vera Apriliani, Jurnal Nasional Teknologi dan Sistem Informasi **4**, 81-88 (2018)
5. Zulmagfir Buako, Lailany Yahya, Novianita Achmad, Jurnal Ilmiah Matematika, Sains dan Teknologi **9**, 62-70 (2021)
6. Cahyaningati, Khalisya Lintang, Resista Vikaliana, Jurnal Ilmiah Ilmu Terapan Universitas Jambi| JIITUJ| **5**, 93-109 (2021)
7. Herlambang, Indra Riksa, Mohamad Nurkamal Fauzan, Rd Nuraini Siti Fathonah, Techno. Com **20**, 430-439 (2021)
8. Serdano, Akbar, Muhammad Zarlis, Dedy Hartama, Seminar Nasional Sains dan Teknologi Informasi (SENSASI) **2**, (2019)
9. Hasugian, Paska Marto, Jurnal Mantik Penusa **18**. (2015)
10. Setiawan, Rendy, R. Gunawan Santosa, Junius Karel Tampubolon, Jurnal Terapan Teknologi Informasi **3**, 95-104 (2019)
11. Agung, Jason Octavianus, Thomas Efendi, Halim Agung, Jurnal Sains Dan Teknologi **5**, 1-7 (2018)
12. Farhan, Syahbani, Septi Andryana, Nur Hayati, JIPI (Jurnal Ilmiah Penelitian dan Pembelajaran Informatika) **5**, 123-132 (2020)
13. Floyd Robert W, *Algorithm 97: shortest path* (Communications of the ACM, 1962) 345
14. Stephen Warshall, Journal of the ACM (JACM) **9**, 11-12 (1962).
15. Richard Bellman, *Quarterly of applied mathematics* **16**, 87-90 (1958)

