

# An algorithm for analyzing the reactive behavior of structural elements of panel buildings

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**Abstract.** The paper considers the reactive aspects of the behavior of structural elements of large-panel housing construction. The problem of determining the position in space of the material point (center of mass) of the objects under study is solved, thus allowing us to evaluate the kinetics of the behavior of structural elements as a whole. Original design schemes are proposed with respect to all spatial axes to determine the dynamic reactions of structural elements' connections under pulsed dynamic action on buildings and structures. A system of local oscillator forces balanced by inertial internal and external influences has been compiled by the method of kinetostatics. The interpretation of the behavior of structural elements of buildings by various oscillators is proposed. This interpretation allowed the periodic harmonics to quantify the change in kinetic energy of structural elements as a result of their impact on each other due to external pulsed dynamic influences. An algorithm for determining dynamic reactions in the junctions of structural elements has been developed and proposed.

**Keywords.** Forecast, position of a point in space, dynamic coupling reaction, calculation scheme, oscillator, mathematical pendulum, main vector, kinetic moment, kinetic energy change

## 1 Introduction

Figure 1 shows a fragment from traces of pulse-dynamic impact [1] on buildings and structures in the form of a branched network of cracks of different opening values [2]. The nature of crack development clearly indicates a multidirectional dynamic effect indicating the complex kinetics of destructive factors. The work raises the question of whether said cracks arise solely as a result of external dynamic action or are the result of a combination of various factors, such as the external and internal inertial reactive behavior of the structural elements when interacting with each other and with the supporting frame of the building.

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**Fig.1.** Linear and torsional character of destruction in the form of vertical, horizontal and inclined cracks from dynamic impacts.

After visual analysis of Figure 1, it becomes obvious that linear and torsional deformations of individual material bodies of structural elements and material points that belong to them are the result of complex external and internal dynamic influences. How is it possible to prevent the development of these multidirectional deformations of structural elements, or - to reduce them? One of such possibilities is the kinetostatic balancing [3] of structures, which can be achieved by balancing the material points belonging to the structural elements as a single system.

## 2 Research methods and models

Due to the impact of various external factors on buildings and structures, their structural elements are subjected to complex trajectories of movement in space, characterized by six degrees of freedom. External vibration effects have different frequencies and amplitudes, which causes corresponding responses of buildings and structures in the form of corresponding reactions of structural elements and buildings and structures themselves as a whole. Figure 2 shows fragments of possible trajectories of movement of individual structural elements, as well as a local calculation scheme for determining the analytical dependencies of the characteristics of the movements of characteristic points belonging to these structural elements.



**Fig. 2.** Scheme of possible vibration vibrations of structural elements relative to each other, and relative to the skeleton of the building.

We present a sequence of actions to achieve the conditions of kinetostatic equilibrium of the objects under consideration.

The external system of forces shown in Fig. 2 will be reduced to a "balanced system of forces" by the method of kinetostatics [4].

To achieve this goal, we propose to add the inertia force  $\vec{\Phi}$  to the system at a certain point on the trajectory of motion at a given time. This will make it possible to achieve balancing of the system, despite various external influences on the structural elements and their characteristic points.

For the proposed considered (material) point of the corresponding structural element of the building, - the balancing force will be a force - equal in modulus to the product of the mass of this point by its corresponding tangential and/or normal accelerations and - directed in the direction opposite to the directions of its accelerations.

Further, the inertial forces "virtually forcibly" applied to the characteristic points of the structural elements will be replaced, respectively, by a force and a pair of forces [5]. We apply them in the center of the reduction, - a force equal to the main vector of the inertia forces, and the moment of the pair of forces – equal to the main moment of the inertia forces relative to the center of the reduction (the center of mass of the structural elements under consideration) in such a way that:

$$\vec{\Phi} = -m\vec{a}_c; \quad (1)$$

$$\vec{M}_c^\Phi = \frac{d\vec{K}_c}{dt} \quad (2)$$

where  $\vec{K}_c$  is the kinetic moment of the elements under consideration relative to the center of mass.

When an element having a plane of material symmetry rotates around an axis passing through the center of mass perpendicular to this plane, the system of forces is reduced to a pair of forces lying in the plane of material symmetry of the element. The moment vector of this pair is defined by the following equation

$$\vec{M}_{Cz}^\Phi = \frac{d(I_{Cz}\vec{\omega})}{dt} = -I_{Cz}\vec{\varepsilon} \quad (3)$$

where  $I_{Cz}$  is the moment of inertia relative to the axis passing through the center of mass.

The algebraic moment of a pair of inertia forces is defined as follows:

$$M_{Cz}^\Phi = -I_{Cz}\varepsilon_z = -I_{Cz}\omega_z = -I_{Cz}\dot{\varphi} \quad (4)$$

With the flat movement of the objects under consideration, having a plane of material symmetry, we will bring the system of inertia forces to a force and to a pair of forces lying in the plane of material symmetry as follows:

$$\vec{\Phi} = -m\vec{a}_c, \quad (5)$$

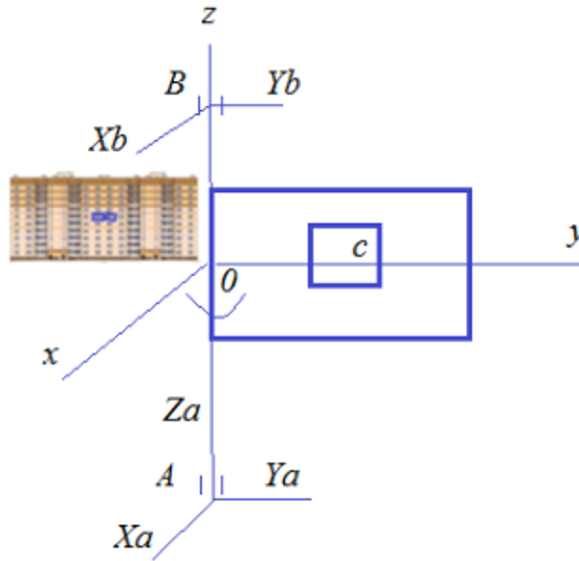
$$M_{Cz}^\Phi = -I_{Cz}\varepsilon_z \quad (6)$$

where  $I_{Cz}$  is the moment of inertia relative to the axis passing through the center of mass perpendicular to the plane of material symmetry.

To determine the dynamic reactions of the interactions of the studied objects with each other and with the supporting frame of the building in the form of "dynamic reactions

of supports" with partial rotation of the studied structural elements and nodes around fixed axes, we will make a virtual calculation scheme, Fig.3.

The essence of this task is to determine the dynamic reactions of interactions between the studied objects and the supporting frame of the building during partial rotation of structural elements and their nodes around fixed virtual axes. As the corresponding relative possible axes of rotation, we will take (assume) vertical and horizontal seams of the corresponding "horizontal and vertical cutting" of buildings and structures:



**Fig. 3.** Scheme for determining dynamic reactions in the attachment points (with a possible alternating rotational movement of the element around the 0Z axis. The Z axis, - is considered a vertical "seam".

Let's denote the structural nodal attachment of the object under study with respect to the assumed axes of rotation by the corresponding points "A" and "B" in Fig. 3,

Having compiled the projection conditions of kinetostatic equilibrium, we obtain a statically definable system of equations of equilibrium of forces and moments (7)

$$\begin{cases} X_{Aдин} + X_{Bдин} + m y_C \ddot{\varphi} + m x_C \dot{\varphi}^2 = 0 \\ Y_{Aдин} + Y_{Bдин} - m x_C \ddot{\varphi} + m y_C \dot{\varphi}^2 = 0 \\ Z_{Aдин} = 0 \\ Y_{Aдин} |z_A| - Y_{Bдин} |z_B| + I_{xz} \ddot{\varphi} - I_{yz} \dot{\varphi}^2 = 0 \\ -X_{Aдин} |z_A| + X_{Bдин} |z_B| + I_{yz} \ddot{\varphi} - I_{xz} \dot{\varphi}^2 = 0 \end{cases} \quad (7)$$

where -  $X_{Aдин}$ ,  $X_{Bдин}$ ,  $Y_{Aдин}$ ,  $Y_{Bдин}$  are the corresponding dynamic reactions of the bonds at the specified points "A" and "B" of the attachment points of the object under study with the rotational form of kinetic behavior from pulsed dynamic influences;  $m$  is the mass of the object under study;  $x_C$  and  $y_C$  are the coordinates of the centers of mass of the objects under study; " $\varphi$ " is the angle of rotation of the object under study relative to the position of the equilibrium state;  $\langle I_{ij} \rangle$  - the corresponding axial moments of inertia relative to the axes assumed in Fig.3.

From the system of equations (7), all unknown values  $X_{\text{Odin}}$ ,  $X_{\text{Vdin}}$ ,  $Y_{\text{Odin}}$ ,  $Y_{\text{Vdin}}$  are expressed in a standard way by jointly solving the above equations.

After performing a minor recoding of the unknowns, we present the code in the Python language for solving the system of equations (7), using the Kramer method:

```
import numpy as np
# Input data
m_list = [1, 2, 3]
Yc = 3 Xc = 4 u = 5 Za = 6 Zb = 7 Lxz = 8 Lyz = 9 f = 10
for m in m_list:
    # Formation of the coefficient matrix
    A = np.array([[1, 1, m*Yc*f, m*Xc*u**2],
                  [0, 0, -m*Xc*f, m*Yc*u**2],
                  [0, 0, 0, 0],
                  [0, 0, y1*abs(Za)-y2*abs(Zb), Lxz*f-
Lyz*u**2],
                  [0, 0, -x1*abs(Za)+x2*abs(Zb), Lyz*f-
Lxz*u**2]])

    # Formation of the vector of the right parts
    b = np.array([0, 0, 0, 0, 0])

    # Calculation of the determinant of the coefficient
matrix
    detA = np.linalg.det(A)

    # Calculation of determinants of matrices with
replacement of columns by a vector of right parts
    det_x1 = np.linalg.det(np.column_stack((b, A[:, 1:], A[:,
2:], A[:, 3:])))
    det_x2 = np.linalg.det(np.column_stack((A[:, 0], b, A[:,
2:], A[:, 3:])))
    det_y1 = np.linalg.det(np.column_stack((A[:, :2], b, A[:,
3:])))
    det_y2 = np.linalg.det(np.column_stack((A[:, :3], b)))

    # Calculation of solutions of a system of equations
    x1 = det_x1 / detA
```

```
x2 = det_x2 / detA
y1 = det_y1 / detA
y2 = det_y2 / detA
# Output of results
print(f"x1 = {x1}, x2 = {x2}, y1 = {y1}, y2 = {y2} при m
= {m}")
```

### **Goals and objectives of the study**

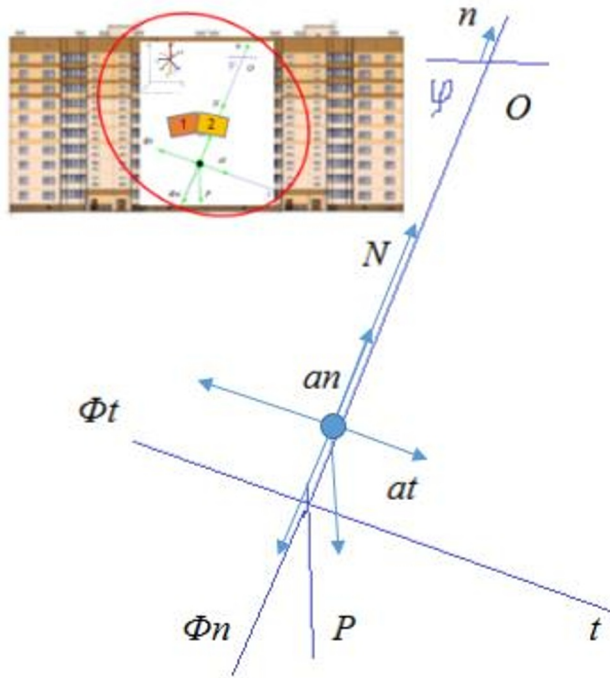
1. Find the equation of motion of a given material point belonging to the object under study at a given location;
2. Find the reactions of the supports in the attachment points of structures from impulse-dynamic effects;
3. Develop an algorithm for solving the problems specified in paragraphs 1 and 2.

### **General provisions for algorithm development.**

Figure 2 shows a possible diagram of the trajectories of vibrations of structural elements of the building relative to each other, as well as possible vibrations of elements relative to the skeleton of the building. The relative axes (0,x,y,z) are introduced and indicated.

We assume that the kinetics of the dynamic behavior of the elements under consideration is similar to the behavior of an oscillator, in the form of a mathematical pendulum, relative to the point "0", with an angle of deviation from the equilibrium position equal to " $\varphi$ ", Fig.4.

A rectangular building object with dimensions ( $l \times h$ ) and mass "m" is represented in Fig.4 in the form of a conditional load "P". The entire structural element in the form of a load "P", - we will consider a material point with a concentrated mass. Attach the load "P" using a conditional weightless rod, - length "L" to the node at point 0, as shown in Fig. 4:



**Fig. 4.** Scheme for determining the dynamic response of the oscillator coupling support at periodic deviation from the equilibrium position

**Algorithm**

**Instruction 1.** Schematically imagine a fragment of an oscillator model with an initial rotation angle equal to " $\varphi$ ", Fig. 4. The rotation angle will be counted from the initial equilibrium position.

**Instruction 2.** Define the forces shown in the diagram as gravity equal to " $\vec{P} = m\vec{g}$ " and the force of internal force in the rod " $\vec{N}$ " as the reaction of the support of the designated rod.

**Instruction 3.** Consider natural forces, the force acting in the direction of the normal to the considered trajectory of motion is equal to " $\vec{\Phi}_n = -m\vec{a}_n$ ". The force acting in the direction of the tangent line to the considered trajectory of motion is equal to " $\vec{\Phi}_t = -m\vec{a}_t$ " of the inertia force. In this case,  $a_n = \frac{v^2}{l}$  and  $a_t = \frac{dv_t}{dt}$ , bearing in mind that  $v = \int a \cdot dt$ ,

Taking into account the indicated, - we will balance the design scheme with the indicated natural forces, Fig. 4

Imagine the desired reaction of the support in the form of  $\vec{N}_R$ , acting in the direction of the line connecting the designated material point "load" of the structural element with its support.

Let's imagine the desired reaction of the support, one of the elements of the set of a balanced system of forces:  $\{\vec{P}, \vec{N}_R, \vec{\Phi}_n, \vec{\Phi}_t\}$ .

Consider the equilibrium equations for the case under consideration:

$$\sum_{k=1}^n F_{kt} = 0, \quad m \cdot g \cdot \cos(\varphi) - \Phi_t = 0 \tag{8}$$

$$\sum_{k=1}^n F_{kn} = 0, \quad N_R - \Phi_n - m \cdot g \cdot \sin(\varphi) = 0 \tag{9}$$

From here:

$$m \frac{dv_{\tau}}{dt} = m \cdot g \cdot \cos(\varphi); \text{ и } N_R = m \cdot g \cdot \sin(\varphi) + \frac{mv^2}{\ell} \quad (10)$$

Let's replace the variable in the differential equation (11) and determine the velocity of the point in question with a mass equal in magnitude to the value "m".

"Cargo", Fig.4 - hangs on the line taken for fastening the structural element to the supporting frame of the building. Along this line, the "normal" force  $N_R$  arises.

$$\frac{dv_{\tau}}{dt} = \frac{dv_{\tau}}{d\varphi} \cdot \frac{d\varphi}{dt} = \omega \cdot \frac{dv_{\tau}}{d\varphi}, \text{ при том, что } \omega = \frac{v_{\tau}}{\ell} \quad (11)$$

As a result of replacing the variable, the differential equation takes the following form

$$\frac{v_{\tau}}{\ell} \cdot \frac{dv_{\tau}}{d\varphi} = m \cdot g \cdot \cos(\varphi), \text{ under the initial condition } v_{\tau}|_{\varphi=0} = 0 \quad (12)$$

The solution is a differential equation under a given initial condition:

$$\int_0^{v_{\tau}} v_{\tau} \cdot dv_{\tau} = g \cdot \ell \cdot \int_0^{\varphi} \cos(\varphi) \cdot d\varphi \quad (13)$$

Integrating, we obtain the desired equation of motion and the analytical expression of the kinematics equation:

$$\frac{v_{\tau}^2}{2} = g \cdot \ell \cdot \sin(\varphi) \quad (14)$$

Using the previously found speed of movement of the material point of the structural element, we will find the modulus of the value  $N_R$  for this solution:

$$N_R = 3 \cdot m \cdot g \cdot \sin(\varphi) \quad (15)$$

It follows from (15) that the considered elements of buildings under these conditions, if they are represented by separate material points, perform complex spatial movements based on harmonic proper and forced oscillations with an oscillation period equal to:

$$T = 2 \cdot \pi \cdot \sqrt{\frac{L}{g}} \quad (16)$$

where "L" is the dimensions of the connection.

### 3 Conclusions

1. The problem of determining the position of the material point of a structural element at a discrete time is solved;
2. A method for determining dynamic coupling reactions under pulsed dynamic action is presented;
3. The dynamics of the reactive behavior of building elements by point and generalized oscillators is shown.
4. The harmonic change in the states of the studied objects from pulsed dynamic influences and the fact that the period of the studied oscillations is directly proportional to the square root of the ratio  $\ll \frac{L}{g} \gg$  are established.
5. It is established that external and internal destructions can be a consequence of inertial and reactive vibration behavior of structural elements.

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