On convergence of nonlinear topological algorithms for calculation of steady modes of electric networks

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Abstract. The paper presents the results of a study of topological algorithms for the formation of a steady state of a complex electrical network, on the convergence of the iterative process. For a power transmission with distributed parameters, consisting of 5 nodes with a voltage of 500 kV, operating on a system of unlimited power, it is shown that there are two solutions and the need to introduce restrictions to obtain real solutions to nonlinear systems of equations. As the regime becomes heavier, the topological method ensures convergence and slow growth of the iterative process compared to the results obtained using the RASTR software package.
Key words: directed graph, tree, chord, current distribution coefficients, nodal voltages.

1 Introduction

The use of distribution coefficients of master currents as parameters of electrical networks makes it possible to simplify the performance of some complex calculations [1]. Therefore, the use of current distribution coefficients from the perspective of the topological theory of electrical networks is of particular interest.

Problems associated with the analytical method for determining the Z matrix of nodal resistances can be solved based on the following. With a known matrix C, it is always possible to find a unique correspondence to the response of the circuit of the electrical network under study to disturbances of the driving currents [2]. Current distribution coefficients, together with branch resistances, quite fully characterize the properties of electrical network circuits, which indicates a certain universality of their application. The complexity of the existing method for determining current distribution coefficients lies in determining the numerators of topological expressions by dividing the network into two parts in order to find two graph trees [3].

For the first time, the exact analytical dependence of the matrix of nodal resistances as a function of the matrix of current distribution coefficients was established by transforming the known matrix equations of the network state in [4]. This paper presents the results of an analysis of the convergence of topological algorithms for the formation of steady-state modes, developed on the basis of the properties of possible graph trees, without dividing the network into two parts [5,6].

2 Materials and methods

The analytical method for constructing the topological matrix of the electrical network of the power system was chosen as the research method. This method is described in detail in [7].

The topological method is applied to a well-known circuit, studied in detail in [2], IEEE test circuits http://energy.komisc.ru/dev/test_cases.

The conducted studies showed that the elements of the matrix of current distribution coefficients of a circuit of arbitrary complexity can be determined based on the natural parameters of the network according to the formula [9]:

$$C_{ij} = \frac{\sum F_{ij}}{\sum F} \tag{1}$$

$F_{ij} -$ is specific tree containing the i-th branch relative to the j-th node; $\sum F -$ is the arithmetic sum of possible trees of the graph.

The steady state of complex networks is described by the matrix equation developed in [6]:

$$\dot{U} = U_0 + C^T Z_B C \hat{U}^{-1} \hat{S} \tag{2}$$

where C is the matrix of current distribution coefficients; Z_B – diagonal matrix of branch resistances; $\hat{U}$ – diagonal matrix of nodal conjugate stresses; $\hat{S}$ – matrix-column of conjugate powers of node loads and generators.

The matrix-column of conjugate powers of nodal loads and generators $\hat{S}$ depends on the nodal voltages:

$$S_k(U_k) = S_{0_k} + j \cdot \alpha_k \cdot |U_k|^2 \cdot 10^{-3} \tag{3}$$

where S_{0_k} – are given fixed values of generation power (load), coefficients α_k are expressed through the given values of capacitive conductivities B, in mSm and B_s, in mSm - the conductivities of shunt reactors.

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3 Results and discussions

For a comparative assessment of the convergence of iterative processes for calculating a steady state in a topological model, let us consider the example discussed in Chapter 3 [2]:

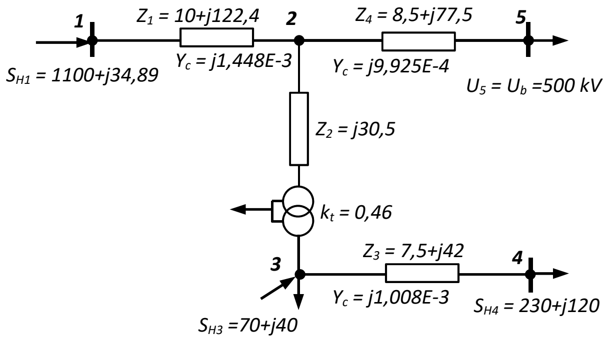

Fig 1. Equivalent circuit of the system under study (resistance in Ohm, conductivity in cm)

A comparison of the results of calculating the steady state performed by the method of simple iteration of the topological model and the Seidel method of the equation of nodal voltages in the form of a current balance is given in table1. It should be noted that the execution time of the proposed program was 10-2 seconds on a regular laptop. First of all, this is due to the fact that in this model there is no need to invert the conductivity matrix, which significantly reduces the number of computational operations.

Table 1. Comparative results of stress calculations with different accuracy (calculation accuracy 10^{-1} - 10^{-3}).

Calculation accuracy	Models and relative difference	Number of iterations	Program execution time (milliseconds)
10^{-1}	1	124	47
	2	95	35
	Δ%	23	25
10^{-2}	1	461	69
	2	328	47
	Δ%	28	31
10^{-3}	1	1313	115
	2	1001	85
	Δ%	23	26

Table 2. Comparative results of stress calculations with different accuracy (calculation accuracy 10^{-1} - 10^{-3}).

	10^{-1}			10^{-2}			10^{-3}		
Re(U1)	404	397	1,70	393	390	0,80	390	387	1,08
IM(U1)	350	351	0,22	351	351	0,09	351	352	0,11
|U1|	534	530	0,86	527	525	0,40	525	522	0,64
arg(U1),°	40,9	41,4	1,35	41,7	42	0,62	42	42,3	0,82
Re(U2)	509	506	0,59	504	503	0,27	503	501	0,35
IM(U2)	113	113	0,18	113	113	0,08	113	113	0,07
|U2|	521	518	0,55	517	515	0,25	515	513	0,32
arg(U2),°	12,5	12,6	0,73	12,7	12,7	0,34	12,7	12,8	0,42
Re(U3)	239	23	0,57	236	236	0,27	236	235	0,33
IM(U3)	44,5	44,5	0,10	44,5	44,6	0,03	44,6	44,6	0,03
|U3|	243	241	0,55	240	240	0,27	240	239	0,46
arg(U3),°	10,6	10,6	0,71	10,7	10,7	0,31	10,7	10,7	0,37
Re(U4)	212	210	0,81	209	208	0,38	208	207	0,48
IM(U4)	2,04	1,8	12,4	1,64	1,51	7,79	1,53	1,36	11,25
|U4|	212	210	0,81	209	208	0,38	208	207	0,48
arg(U4),°	0,55	0,49	11,3	0,45	0,41	7,56	0,42	0,37	10,71

Comparative table of calculations: 1-Seidel method for equations of nodal voltages in the form of current balance, 2-method of simple iteration of the topological model, Δ-relative difference in percentage.

Let us now consider the 5-node test circuit rucase_5_4.txt. The equivalent circuit has the form shown in Fig. 2.

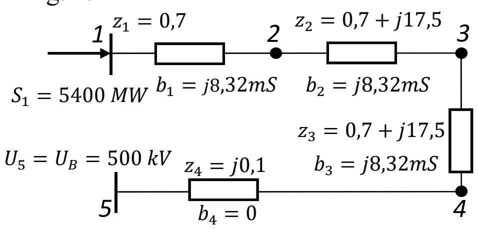

Fig. 2. IEEE test 5-node circuit rucase_5_4.txt.

The initial and calculated data are presented in the following tables.

Table 3. Calculated and initial data for nodes of the test circuit rucase_5_4.txt.

Nodes №	U nom	Voltage		Load power		Generation power	
	kV	phase, de	module, kV	P, MW	Q, MVar	P, MW	Q, MVar
1	500	68.73	484.5	0.0	0.00	5400	0.00
2	500	45.05	482.0	0.0	0.00	0.00	0.00
3	500	21.97	489.3	0.0	0.00	0.00	0.00
4	500	0.12	499.9	0.0	0.00	0.00	0.00
5	500	0.00	500.0	0.0	0.00	0.00	457

Table 4. Initial data for the branches of the test circuit rucase_5_4.txt.

Branch no.	start	end	R, Ohm	X, Ohm	B, mSm (cap+,ind-)
1	1	2	0.7000	17.5000	8.3200
2	2	3	0.7000	17.5000	8.3200
3	3	4	0.7000	17.5000	8.3200
4	4	5	0.0000	0.1000	0.0000

The calculations performed using both methods coincided exactly to the third decimal place. It should be noted that in this case there are several solutions to the vector equation (2), so it is necessary to take into account technical restrictions such as $|U_1| \leq 550.$

Moreover, the execution time and number of iterations, depending on the accuracy of the solution, are as follows:

Table 5. Calculated data for a 5-node circuit rucase_5_4.txt

Calculation accuracy	Number of iterations	Program execution time (milliseconds)
0,1	4	22
0,01	8	28
0,001	16	32
0,0001	18	48
1E-05	24	64

The required number of iterations for a 5-node scheme turned out to be greater than expected. This is because in this case we are considering a case close to the maximum power transmission power, in which additional “imaginary” solutions of the nonlinear system arise, to which the iteration converges very quickly, almost regardless of the required accuracy of the solution (no more than 5-6 iterations).

As noted above, to exclude these imaginary solutions, additional conditions are imposed on the solution to limit the voltage module to 525 kV. Therefore, the number of iterations increases. If we reduce the power transfer by 20%, that is, instead of 5400 MW we take, for example, 4400 MW, then the iteration converges to the desired solution quite quickly. Below are the calculation results for a generation power of 4400 MW.

Table 6. Calculated data for a 5-node circuit rucase_5_4.txt with a generation capacity of 4400 MW.

Calculation accuracy	Number of iterations	Program execution time (milliseconds)
0,1	4	8
0,01	8	8
0,001	5	12
0,0001	5	14
1E-05	6	18

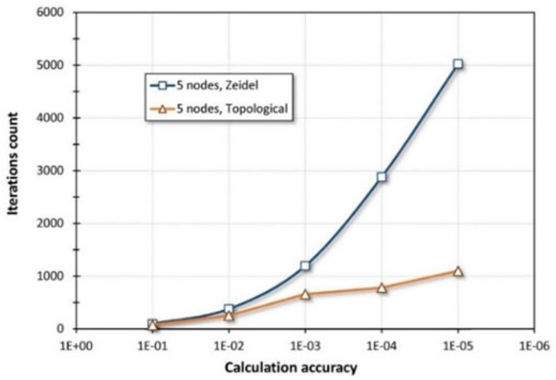

Fig. 3. Dependence of the number of iterations on the calculation accuracy in semi-logarithmic coordinates.

As can be seen from the graphs presented in Figure 3, calculations by the Seidel method become significantly more complicated with increasing accuracy. At the same time, the topological model method proposed by the authors shows a slight increase in the complexity of calculations with increasing accuracy, which is an extremely promising method for calculating complex circuits with high accuracy.

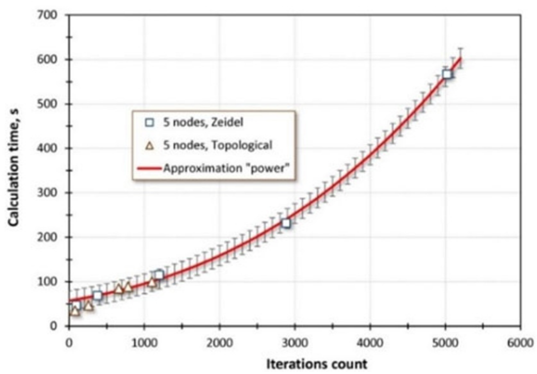

Fig. 4. Dependence of program execution time on the number of iterations.

Figure 4 shows the general dependence of the increase in calculation duration with an increase in the number of iterations for the 5-node circuit presented in Figure 2. As can be seen from the figure, with an increase in the number of iterations, the calculation time increases according to a power law, which is shown by the solid line.

The equation for the approximating curve was obtained according to the recommendations [10]:

$$T = 30 + 1.05 \cdot 10^{-9} \cdot (Ci + 2970)^3 (EQAdd.01)$$

This type of dependence allows us to describe the ratio of the time of the number of iterations with the coefficient of determination $R^2 = 0.999$. It should be noted that using the standard “Power” trend line in MS Excel gives the coefficient of determination value $R^2 = 0.88$, and for the “Exponential” trend line $R^2 = 0.93$, which is significantly lower than the value we obtained.

The vertical dashes on the solid line in Figure 3 show the standard error of approximation. As can be seen from the figure, it does not exceed 5%, which also confirms the correctness of the chosen form of the equation.

Thus, it is shown that the topological model makes it possible to increase the speed of calculations several times for complex circuits with high calculation accuracy compared to the Seidel method.

To confirm the conclusion obtained, the authors performed additional calculations of various electrical circuits, including more complex ones in composition and number of nodes.

Next, we present the results of applying the developed topological model to various test circuits.

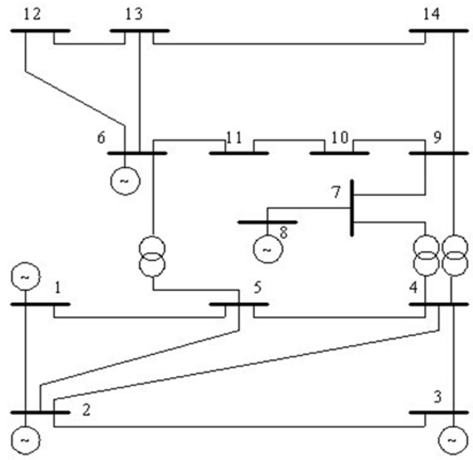

Fig. 2. IEEE test 14-node circuit

Table 7. Initial data for the branches of the test circuit

Branch no.	start	end	R, Ohm	X, Ohm	B, mSm (cap+, ind-)	k_t
1	1	2	10,25	31,30	0,0998	-
2	1	5	28,58	117,99	0,093	-
3	2	3	24,86	104,73	0,0828	-
4	2	4	30,74	93,27	0,0643	-
5	2	5	30,13	91,98	0,0654	-
6	3	4	35,45	90,47	0,0242	-
7	4	5	7,06	22,28	0	-
8	4	7	0	105,81	0	0,51125
9	4	9	0	276,26	0	0,516
10	5	6	0	115,80	0	0,53648
11	6	11	12,56	26,30	0	
12	6	12	16,25	33,83	0	
13	6	13	8,75	17,23	0	
14	7	8	0	23,29	0	-
15	7	9	0	14,55	0	-
16	9	10	4,21	11,17	0	-

Continuation of Table 7. Initial data for the branches of the test circuit

Branch no.	start	end	R, Ohm	X, Ohm	B, mSm (cap+, ind-)	k_t
17	9	14	16,81	35,76	0	-
18	10	11	10,85	25,40	0	-
19	12	13	29,22	26,43	0	-
20	13	14	22,6	46,02	0	-

Table 8. Comparative results of calculations.

Nod. no	Voltage calculation in the RASTR program		Voltage calculation in the topological model	
	phase, deg	module, kV	phase, deg	module, kV
1	0	243,8	0	243,8
2	4,983	240,35	4,973	240,141
3	12,725	232,3	12,73	231,813
4	10,313	234,064	10,291	233,361
5	8,774	234,488	8,772	234,375
6	14,221	123,05	14,056	124,538
7	13,36	122,075	13,424	120,36
8	13,36	125,35	13,424	126,882
9	14,939	121,432	15,152	120,033
10	15,097	120,863	15,284	120,878
11	14,791	121,544	14,83	120,802
12	15,076	121,347	15,025	119,867
13	15,156	120,794	15,152	119,395
14	16,034	119,086	16,2363	118,503

As we can see, the maximum deviation of the voltage modulus is 1.4%, and it is good results.

Table 9. Calculated data for the circuit

Calculation accuracy	Number of iterations	Program execution time (milliseconds)
0,1	6	22
0,01	8	24
0,001	10	28
0,0001	12	33
1E-05	13	38

When considering networks with a lot of nodes with possible random dynamic changes, associated, for example, with various types of emergency failures of nodes, it is necessary to consider random graphs. They can be described and studied using pseudo-finite models of standard graph theory. General laws for well-defined systems can be investigated using statistical and model-theoretic methods. From a model-theoretic point of view, one can approach the approximation [11]. The family of such pseudo-finite theories, as well as their topological properties, were studied in [12].

4 Conclusion

The considered iterative process has reliable and faster convergence than the Gauss-Seidel method using a matrix of nodal conductivities. The proposed method, unlike the method using Zy, does not require determining and storing a matrix of nodal resistances. A feature of the new topological method for calculating the mode of an electrical network is that the current distribution coefficients in a complex circuit can be determined at the stage of generating initial data on the network topology, which significantly reduces the computational costs of real calculations of a complex electrical network.

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